Approved by the Undergraduate Council

Sharon Gill

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Date: 2011.01.13 13:53:47 -05'00'

To: The quantitative foundations vetting committee
From: Jakayla Robbins, Director of Service Courses, Department of Mathematics
Subject: Proposal to include MA 123, Elementary Calculus and Its Applications in the new General Education Program.

The Department of Mathematics requests that MA 123, *Elementary Calculus and Its Applications*, be approved as a Quantitative Foundations course for the General Education program. Currently, MA 123 is a premajor requirement for many majors including majors in biological sciences, agriculture, and the College of Business and Economics. We anticipate that most of the students with these majors will continue to take MA 123 because functional relationships and rates of change are fundamental concepts in the sciences and economics. Calculus is the first collegiate course that fully develops these concepts. (We note that we are developing a more rigorous Calculus course for biology majors who want a career that is more mathematically demanding. We anticipate that the majority of biology majors will continue to take MA 123.)

We believe that the current syllabus for MA 123 does address learning outcomes 1 and 2 for the Quantitative Foundations course, but it does not address the information literacy requirement for all General Education courses. Consequently, we are submitting a slightly modified version of the current syllabus for approval by the vetting committee. The modified version of the syllabus includes two semester projects which are new to the course.

We believe that MA 123 satisfies Learning Outcome 1 for many reasons. For example:

(1) Students use function notation to describe the relationship between the input of a function and its output.
(2) Students graph functions.
(3) Students estimate limits and derivatives by looking at tables of functional values.
(4) Students begin to understand continuity and differentiability by examining graphs.
(5) Students begin to understand how to find the slope of a tangent line by estimating slopes of secant lines.
(6) Students begin to understand how to find the area under a curve by using rectangles to estimate the area under the curve.

We believe that MA 123 satisfies Learning Outcome 2 for many reasons. For example:

(1) Students are asked to create models for everyday problems that involve exponential growth and decay. They must use these models to solve problems.
(2) Students are asked to create models for everyday problems that involve compound interest. They must use these models to solve problems.
(3) Students must solve quiz problems at least once each week. Students show their work on quiz problems.
(4) In the new project about population growth, students will construct a model, use it to make estimates, and compare their estimates to real data.
(5) Although instructors may not use the word “converse” in class, students are asked to recognize that the converse of an implication is not equivalent

to the original implication. For example, if a function is differentiable then it must be continuous, but a continuous function need not be differentiable.

(6) Students use information obtained from the first and second derivatives of a function to sketch graphs.
(7) Students use tabular data to justify estimations of limits.
(8) Students are asked to make conjectures in the new spreadsheet project based on numerical evidence.
(9) Students use the methods of Calculus to solve optimization problems. In this process, they must interpret the results of the first derivative test to determine maximum and minimum values. Moreover, they must distinguish between the input and output values of the original function and its derivative, and they must determine the meaning of each of these values in relation to an everyday problem.
(10) Students use the methods of Calculus to solve related rates problems. In this process, they must do a unit analysis of derivatives and correctly represent everyday problems in the language of Calculus.

We believe that the modified version of MA 123 will satisfy the information literacy requirement because:

(1) The new project on population growth requires students to read information and watch videos that explain how to choose credible sources. This information was provided by UK Libraries.
(2) The new project on population growth asks students to find population data and to explain why their source is credible.
(3) The new spreadsheet project asks students to find an Excel command, to learn how to use the command, and to use the command to solve a problem.

The remainder of this document consists of several attachments. The first attachment is a modified version of the current syllabus. The syllabus is web based, so it is not easy to include it here. Each instructor has a separate syllabus with some specific information about his or her section. This syllabus refers to the main syllabus which is relevant for all students in MA 123. The first portion of the syllabus is a sample syllabus for an individual instructor. The main syllabus follows. This syllabus is a web page with frames. The left-hand side of the page contains several links. The right-hand side of the page changes depending on which link you have selected. Printing both sides of the page simultaneously does not work well. Consequently, we first included the menu that is always visible on the left-hand side of the page. After this, we printed the links. We did not print the links for the Mathskeller or the Study because these are not directly related to the course content. After the syllabus, we have included one set of lecture notes as an example. Finally, we have attached the two projects that were created to satisfy the information literacy requirement. The first project investigates limits and derivatives using a spreadsheet. This spreadsheet contains two tabs. The first tab is a tutorial. The second tab contains answers to the tutorial. The final tab contains the project itself. This project was designed because the CRAFTY project (Curriculum Reform Across the First Two Years) of the Mathematical Association of America suggests that any calculus class targeted at biology majors and any calculus class targeted at business majors should contain a component that asks students to use a spreadsheet. A significant portion of the students who take MA 123 are either biology or business majors. The second project is a project about population growth. The exponential and logistic growth models introduced in this project appear in applications in the sciences and economics.

Instructor Syllabus

MA 123 Sections 001 and 002, Elementary Calculus, Semester 2XXX

Instructor: Sample Instructor, Ph.D.
Department of Mathematics
University of Kentucky

Office: 1902 Patterson Office Tower (1902 POT)
Telephone: (999) 999-9999
(preferred method for reaching Sample Instructor) **E-mail:** *instructor at ms.uky.edu*

Office Hours (for Sample Instructor)**:** Monday 9:00 - 9:55 am (Mathskeller)
Monday 10:00 - 11:00 am (POT 1902)
Wednesday 9:00 - 10:00 am (POT 1902)
No appointment is necessary at the times listed above. Additional meeting times can be made by appointment and are welcome.

Lecture: Sections 001 and 002, MWF 12:00 - 12:50 pm, CP 153

Teaching Assistants:
Tea Aone (*ta1 at uky.edu*)
Tea Atwo (*ta2 at uky.edu*)

Office Hours (for the teaching assistants)**:**
Tea Aone: *Monday and Friday 1:00 pm -3:00 pm, 4:00 pm -5:00 pm and Wednesday 1:00 pm - 3:00 pm* in the Mathskeller
Tea Atwo: *Tuesday and Thursday 1:00 pm -3:00 pm, 4:00 pm -5:00 pm and Wednesday 1:00 pm - 3:00 pm* in the Mathskeller

Recitations:
Section 001, Tea Aone, T 1:00 - 2:15 pm, CP 397
Section 002, Tea Atwo, R 1:00 - 2:15 pm, CP 397

Undergraduate Assistants: Undergrad One (*undergrad1 at uky.edu*) and Undergrad Two (*undergrad2 at uky.edu*)

Office Hours (for the undergraduate assistants)**:**
Undergrad One: *Tuesday 11:00 am - 1:00 pm* in the Mathskeller
Undergrad Two: *Tuesday 11:00 am - 12:00 pm and Friday 2:00-3:00 pm* in the Mathskeller

The **main syllabus** for Ma 123 can be found at www.ms.uky.edu/~ma123/
Homework for the class can be found at https://www.mathclass.org/mc/default.aspx

Grading policy: The grading policy for all sections of the course is described in the main syllabus. A portion of your grade (4%) is based on your quiz scores. You can earn up to 500 points for the course. Of these 500 points, 20 points are determined by your quiz scores. A short 5 minute quiz will be given approximately two times per week. Each quiz score will be between 0 and 2. The number of points that you have earned toward the quiz portion of your final grade can be calculated using the following formula.

(Average Quiz Score)/2 * 20

You will need to purchase a set of 3 by 5 notecards for the quizzes and bring them with you to class each day.

Classroom behavior: I expect students to come to class on time and show respect for fellow students. That includes, but is not limited to the following. Do not talk unnecessarily during class. The lecture class is large and conversations among students will disturb others who are trying to listen to the lecture. Do not leave class early, since that can be interfere with others who are trying to listen to the lecture. You may not use the following items during class: MP3 players, cell phones, laptop computers, other electronic devices not related to this math class. You should not read the newspaper or work on assignments from other classes during this math class. You should not work on anything unrelated to the current lecture during class.

Advice: Form good study skills from the start. Come to class. Read the text and do the homework. Do not fall behind. It is often hard to catch up in a math class after falling behind. If you are having trouble, then seek help without delay. There are many resources available in the Mathskeller and in the The Study. Find classmates to study with. Go to office hours. Talk to me during office hours. Talk to or send e-mail to the teaching assistant. Talk to or send e-mail to the undergraduate assistants, if necessary. Let me know if you are having problems or if you think that there is a problem with the course.

Main Syllabus

MA 123
Semester 20xx

MA 123 (Elementary Calculus)

Time & Location: click **here**.

Texts:
The text listed in #1 below is the official text for the course. It is very readable and has many worked out examples. The set of notes listed under #3 below is available for free online. The fourth text listed below is a good study guide for students.

1. **Calculus** by Elliot Gootman. The book is published by Barron's, and it will be the primary text for the course. The book can be purchased from the bookstores or online. We shall cover the first ten (10) chapters of this book.
2. **Supplemental Handouts and/or Lecture Notes**. (Click **here** to download.)
3. **A Brief Introduction to Calculus** by Robert Molzon. (Click **here** to download.) This is a set of notes that you may find useful for basic concepts. You might want to download the notes and print a copy. The initial part of the course will review algebra, and these notes should be very useful for that review.
4. **How to Ace Calculus: The Streetwise Guide** by C. Adams, J. Hass, and A. Thompson. This inexpensive book might be a useful guide to study methods for calculus and math in general.

Course Content:
This course introduces differential and integral calculus. We will cover each of the topics from the first ten chapters of the text, Calculus, by Gootman. Thes topice include, functions, rates of change, limits, derivatives, extreme values, the Mean Value Theorem, the definite integral, antiderivatives, the Fundamental Theorem of Calculus, and application problems. All of these topics are covered in the online homework sets.

Student Learning Outcomes:
By the end of the semester, it is expected that students will

1. demonstrate proficientcy with number sense and with functional relationships between

two or more sets of variable values and also relate different representations of such relations.

2. apply funcamntal elements of mathematical knowledge to model and solve problems drqwn from real life.

Goals:

Your main goal should be to learn the material well enough so that you can use calculus in an applied context such as business or social science. It is virtually impossible to learn mathematics without actively taking part in the learning. To understand what this means, consider the impossibility of learning to play tennis by listening to someone describe how to play tennis. You will not learn the material in this course by listening to the lectures, and thinking to yourself - "Yes, I understand that". You must work the problems and make mistakes before you will begin to learn. The instructor's task is that of an assistant to help you learn as much of the material as you desire.

In this course it will not be sufficient to memorize an algorithm for doing specific types of problems. You will be expected to understand the material well enough so that you can work problems similar to, but not identical to the ones we work in class and the ones you encounter in the homework.

Prerequisites:

You should have a strong understanding of college algebra and an ACT score of at least 26 or a score of 70% on the placement exam. If you have a weak algebra background it is essential that you immediately brush up on this prerequisite. Most students who do not do well in calculus, find that the required algebra is the major roadblock.

Corrections to: *corso@ms.uky.edu*

Tentative Course Schedules

Click here if you are enrolled in a MWF class.

Click here if you are enrolled in a TR class.

Other important dates:

W 1/13	First day of classes
M 1/18	Martin Luther King birthday (academic holiday)
W 1/20	Last day to add a class
W 2/3	Last day to drop a class without a grade
M 3/15 - S 3/20	Spring break (academic holidays)
F 4/2	Last day to withdraw from a class
F 4/30	Last day of classes
R 5/6	Final exam

Tentative Course Schedule - MWF Classes

Date	Description
W 1/13	Introduction/Algebra Review (Chapter 1)
F 1/15	Algebra Review (Chapter 1)
M 1/18	*Martin Luther King, Jr. Birthday Celebration - no class*
W 1/20	Rates of Change (Chapter 2)
F 1/22	Rates of Change (Chapter 2)
M 1/25	Rates of Change (Chapter 2)
W 1/27	Limits (Chapter 3)
F 1/29	Limits (Chapter 3)
M 2/1	Limits (Chapter 3)
W 2/3	Computing some Derivatives (Chapter 4)
F 2/5	Computing some Derivatives (Chapter 4)
M 2/8	Review for Exam 1
W 2/10	Review for Exam 1; EXAM 1, 5-7 pm
F 2/12	Formulas for Derivatives (Chapter 5)
M 2/15	Formulas for Derivatives (Chapter 5)
W 2/17	Formulas for Derivatives (Chapter 5)
F 2/19	Higher Derivatives (Chapter 5)
M 2/22	Exponential and Logarithmic Functions (Supplement)
W 2/24	Exponential and Logarithmic Functions (Supplement)
F 2/26	Exponential and Logarithmic Functions (Supplement)
M 3/1	Extreme Values and Mean Value Theorem (Chapter 6)
W 3/3	Extreme Values and Mean Value Theorem (Chapter 6)
F 3/5	Curve Sketching and Concavity (Chapter 6)
M 3/8	Review for Exam 2
W 3/10	Review for Exam 2; EXAM 2, 5-7 pm
F 3/12	Curve Sketching and Concavity (Chapter 6)
M 3/15	*Spring Break - no class*
W 3/17	*Spring Break - no class*
F 3/19	*Spring Break - no class*
M 3/22	Word Problems (Chapter 7)
W 3/24	Word Problems (Chapter 7)
F 3/26	Word Problems (Chapter 7)
M 3/29	The Idea of the Integral (Chapter 8)
W 3/31	The Idea of the Integral (Chapter 8)
F 4/2	The Idea of the Integral (Chapter 8)
M 4/5	Computing some Integrals (Chapter 9)
W 4/7	Computing some Integrals (Chapter 9)
F 4/9	Computing some Integrals (Chapter 9)

M 4/12	Review for Exam 3
W 4/14	Review for Exam 3; EXAM 3, 5-7 pm
F 4/16	Fundamental Theorem of Calculus; Antiderivatives (Chapter 10)
M 4/19	Fundamental Theorem of Calculus; Antiderivatives (Chapter 10)
W 4/21	Fundamental Theorem of Calculus; Antiderivatives (Chapter 10)
F 4/23	Fundamental Theorem of Calculus; Antiderivatives (Chapter 10)
M 4/26	Review for Final Exam
W 4/28	Review for Final Exam
F 4/30	Review for Final Exam
R 5/6	FINAL EXAM, 6-8 pm

Tentative Course Schedule - TR Classes

Date	Description
R 1/14	Introduction; Algebra Review (Chapter 1)
T 1/19	Algebra Review (Chapter 1); Rates of Change (Chapter 2)
R 1/21	Rates of Change (Chapter 2)
T 1/26	Rates of Change (Chapter 2); Limits (Chapter 3)
R 1/28	Limits (Chapter 3)
T 2/2	Limits (Chapter 3); Computing Some Derivatives (Chapter 4)
R 2/4	Computing Some Derivatives (Chapter 4)
T 2/9	Review for Exam 1
W 2/10	EXAM 1, 5-7 pm
R 2/11	Formulas for Derivatives (Chapter 5)
T 2/16	Formulas for Derivatives (Chapter 5)
R 2/18	Higher Derivatives (Chapter 5)
T 2/23	Exponential and Logarithmic Functions (Supplement)
R 2/25	Exponential and Logarithmic Functions (Supplement)
T 3/2	Extreme Values and Mean Value Theorem (Chapter 6)
R 3/4	Extreme Values and Mean Value Theorem (Chapter 6)
T 3/9	Review for Exam 2
W 3/10	EXAM 2, 5-7 pm
R 3/11	Curve Sketching and Concavity (Chapter 6)
T 3/16	*Spring Break - no class*
R 3/18	*Spring Break - no class*
T 3/23	Word Problems (Chapter 7)
R 3/25	Word Problems (Chapter 7)
T 3/30	The Idea of the Integral (Chapter 8)
R 4/1	The Idea of the Integral (Chapter 8)
T 4/6	Computing some Integrals (Chapter 9)
R 4/8	Computing some Integrals (Chapter 9)
T 4/13	Review for Exam 3
W 4/14	EXAM 3, 5-7 pm
R 4/15	Fundamental Theorem of Calculus; Antiderivatives (Chapter 10)
T 4/20	Fundamental Theorem of Calculus; Antiderivatives (Chapter 10)
R 4/22	Fundamental Theorem of Calculus; Antiderivatives (Chapter 10)
T 4/27	Review for Final Exam
R 4/29	Review for Final Exam
R 5/6	FINAL EXAM, 6-8 pm

Course Policies

Grading

You will be able to obtain a maximum of 500 points in this class, divided as follows:

Exam 1, 90 points (18% of your grade)
Exam 2, 90 points (18% of your grade)
Exam 2, 90 points (18% of your grade)
Final exam, 90 points (18% of your grade)
Project 1, 20 points (4% of your grade)
Project 2, 20 points (4% of your grade)
Homework, 80 points (16% of your grade)
Quizzes, 20 points (4% of your grade)

There will be two projects in this class. Each project is worth 20 points. These projects will be due at the beginning of class on their due dates.

The Homework score will be computed as follows. There are 250 homework problems in the course, but the homework grade will be based on 225 problems. Thus, if you answer x homework problems correctly, your homework score will be (x/225) times 80. (225 correct homework problems will give you 80 points for the homework.) Note that you if you answer more than 225 homework problems correctly, then you could end up with more than 100 points for the "Homework" score.

The quiz score will be at most 20 points. Your instructor will decide exactly how to award those points based on your in-class quizzes. The instructor will explain how you can calculate your quiz score in the syllabus for your section of MA 123.

Your final grade for the course will be based on the total points you have earned as follows.

A: 450-500
B: 400-449

C: 350-399

D: 300-349

E: 0-299

The grading scale might be adjusted at the end of the semester on a course-wide basis (in other words, every instructor will use the same grading scale). You will be guaranteed the above letter grade if your score falls within the given range, but the minimum score for each letter grade might be lowered.
Undergraduate students will be provided with a midterm grade by the midterm date. The midterm grade will be calculated based on the work you have completed by the date your instructor calculates midterm grades and the criteria in this syllabus.

Attendance
Attendance in MA123 is mandatory. Your instructor will inform you how s/he keeps track of your attendance. You should inform your instructor of any excused absence in a timely manner and provide documentation verifying that the absence is excused. It is almost always possible to inform your instructor of an excused absences before missing class. Only students who are seriously ill or find themselves in an unexpected emergency situation should inform the instructor after the absence. You will be permitted to make up work that you missed because of an excuesed absence. You will not be permitted to make up work that you missed because of an unexcused absence. For more information about excused absences, see **Section 5.2.4.2 of Student Rights and Responsibilities**.

Students who have university excused absences or who have university-scheduled class conflicts with uniform examinations may arrange with their instructor to take the exam at an alternate time. Generally these make-up exams will be scheduled on the day after the regularly scheduled exam. The time and room will be announced later. Work-related conflicts are neither university excused absences nor university-scheduled absences.

Academic Honesty
Cheating or plagiarism is a serious offense and will not be tolerated. It will be thoroughly investigated, and might lead to failure in the course or even to expulsion from the university. See
http://www.uky.edu/StudentAffairs/Code/part2.html
(Sections 6.3.1 and 6.3.2) for information on cheating, plagiarism, and penalties.

A summary of recent changes to rules on cheating can be found at the Academic Ombud website:
http://www.uky.edu/Ombud Take special care not to plagiarize when you are completing the projects for this course. You may work with other students to help you solve the problems in the projects, but you should give them credit in your final assignment. Moreover, if you work with other students, you should be certain to write your answers on your own. Make sure that you are by yourself when you are writing your answers and you are not looking at another person's work.

Academic Accommodations due to Disability

If you have a documented disability that requires academic accommodations, please see your instructor as soon as possible during scheduled office hours. In order to receive accommodations in this course, you must provide your instructor with a Letter of Accommodation from the Disability Resource Center (Room 2, Alumni Gym, 257-2754, email address jkarnes at email.uky.edu) for coordination of campus disability services available to students with disabilities.

Sections and Instructors

If you contact your instructor by email, be sure to place "ma123" in the subject line. An email without this subject heading may be deleted by spam filters.

Instructor's Name	**Sections** (#, time, location)	**E-mail Address**
Sample Instructor	001-002 - MWF 12:00pm-12:50pm - CP 153	instructor@ms.uky.edu
Tea Aone (recitation)	001 - T 1:00pm-2:15pm - CP 397	ta1@ms.uky.edu
Tea Atwo (recitation)	002 - R 1:00pm-2:15pm - CP 397	ta2@ms.uky.edu

Lecture Notes

Detailed notes with the plan for each chapter (goals, main facts and problems to be discussed in class) have been written in order to assist you throughout the course. They will be used as a primary means of instruction in some of the sections. They can be downloaded here:

Chapter 1: **Equations, functions and graphs**
lecture notes **practice/review problems**
notes with answers **answers to practice/review problems**

Chapter 2: **Change, and the idea of the derivative**
lecture notes **practice/review problems**
notes with answers **answers to practice/review problems**

Chapter 3: **The idea of limits**
lecture notes **practice/review problems**
notes with answers answers to practice/review problems

Chapter 4: **Computing some derivatives**
lecture notes **practice/review problems**
notes with answers answers to practice/review problems

Chapter 5: **Formulas for derivatives**
lecture notes **practice/review problems**
notes with answers answers to practice/review problems

Supplement: **Exponential and logarithmic functions**
lecture notes **practice/review problems**
notes with answers answers to practice/review problems

Chapter 6: **Extreme values, the Mean Value Theorem, and curve sketching**
lecture notes **practice/review problems**
notes with answers answers to practice/review problems

Chapter 7: **Word problems**
lecture notes practice/review problems
notes with answers answers to practice/review problems

Chapter 8: **The idea of the integral**
lecture notes practice/review problems
notes with answers answers to practice/review problems

Chapter 9: **Computing some integrals**

lecture notes | practice/review problems
notes with answers | answers to practice/review problems

Chapter 10: **Formulas for integrals: integrals, antiderivatives and the Fundamental Theorem of Calculus**

lecture notes | practice/review problems
notes with answers | answers to practice/review problems

Homework

The course uses an online homework system called WebClass , and your homework is graded and recorded by the system. You can get to WebClass by going to the link **https://www.mathclass.org** . Information about WebClass is found at the link **webclass_information.html** . **Before you try to login to WebClass, be sure to read the information on this page**. Be sure you have Cookies and Popups enabled on your browser. After you login, select the web homework link. This will take you to your MA123 class where you do your homework.

Each student has an individual, Personal Version of the web-based homework assignments to work and submit. For each problem set there is also a Common Version similar to the personal version. Everyone gets the same Common Version. Problems on the Common Version are the ones most likely to be discussed in class. **You are expected to solve the Personal Version on your own without help**.

You may attempt a problem as many times as you like. Only your final (and hopefully correct!) answer will be recorded for your homework grade. Additional attempts at a problem need not be made in the same online session, so you can reattempt the problem after getting help from your instructor, in the Mathskeller, or in the Study at the Complex Commons. If you submit the correct answer to a problem before the due date, you receive full credit for the problem.

Caution: The homework policy is generous in that you can attempt a homework problem as many times as needed. After each attempt, the computer tells you whether your answer is correct or not. During an exam, there is no immediate feedback. You must answer the question correctly the first time. In order to be well prepared for the exam, you must be able to work a problem correctly the first time without receiving feedback that a mistake has been made. Practice the homework problems enough so that you can do this. Receiving a high score in the homework might not be enough preparation for the exams if many of the homework problems were worked correctly only after multiple attempts.

If you submit an incorrect answer to a homework problem, the first thing to check is the syntax you used to submit the answer. A typo will obviously result in an incorrect answer. If you check the syntax carefully, and your answer is still incorrect, go back and rework the problem. It is often better to work other problems first, since it is quite easy to make the same error over and over. If after several attempts, you do not get a correct answer, then get help from your instructor or from a tutor in the Study or Mathskeller. Although the answers to all problems have been checked, it is still possible that there is an error in the system.

Homework will generally be due twice a week at midnight. The due dates are indicated on the homework sets. It is your responsibility to check these dates. DO NOT WAIT UNTIL THE EVENING OF THE DUE DATE to do the homework. No homework can be accepted late. If you miss class for a University trip, you must complete your assignments ahead of the due date.

In addition to the online homework, your instructor might give quizzes during class. If this is the case, your instructor will inform you how these quizzes will contribute to the 100 points in the homework category for your section. **Note that class attendance is mandatory for MA123. You will not be allowed to make up missed quizzes unless you have a valid university excuse**. If you anticipate an excused absence you must notify your instructor in writing two weeks before the absence.

Homework Due Dates

Due Date	Homework
Thursday 1/21	HW 00: Algebra Review (1)
Monday 1/25	HW 01: Algebra Review (2)
Thursday 1/28	HW 02: Rates of Change (1)
Monday 2/1	HW 03: Rates of Change (2)
Thursday 2/4	HW 04: Limits (1)
Monday 2/8	HW 05: Limits (2)
Wednesday 2/10	Exam 1
Monday 2/15	HW 06: Computing Dertivatives (1)
Thursday 2/18	HW 07: Computing Dertivatives (2)
Monday 2/22	HW 08: Formulas for Derivatives (1)
Thursday 2/25	HW 09: Formulas for Derivatives (2)
Monday 3/1	HW 10: Exponential and Logarithmic Functions
Thursday 3/4	HW 11: Exponential Growth and Decay
Monday 3/8	HW 12: MVT (1)
Wednesday 3/10	Exam 2
Monday 3/22	HW 13: MVT (2)
Thursday 3/25	HW 14: Concavity
Monday 3/29	HW 15: Word Problems (1)
Thursday 4/1	HW 16: Word Problems (2)
Monday 4/5	HW 17: Idea of the Integral (1)
Thursday 4/8	HW 18: Idea of the Integral (2)
Monday 4/12	HW 19: Computing some Integrals (1)
Wednesday 4/14	Exam 3
Monday 4/19	HW 20: Computing some Integrals (2)
Thursday 4/22	HW 21: Fundamental Theorem of Calculus (1)
Monday 4/26	HW 22: Fundamental Theorem of Calculus (2)
Thursday 4/29	HW 23: Course Review

Web Homework Instructions

Students who are registered for MA 109, MA 123, MA 113, MA 162 and select sections of MA 110, MA 114 and MA 213 will use an online homework called *WebClass*. Note that the url for WebClass is **http://www.mathclass.org**. Students enrolled in these classes will have an account created for them. PLEASE DO NOT CREATE AN ACCOUNT UNLESS ASKED TO DO SO BY YOUR INSTRUCTOR. If you create an account, you will not be able to add your class to this account.

1 System requirements

We have tested *WebClass* on Windows computers in the computer labs across campus . If you note a problem at one of these labs, please submit a help request through. We are not able to provide support for computers owned by individuals. However, we have found that works on Windows XP and Windows Vista with the browsers Internet Explorer 7 and Firefox 2.0. Internet Explorer requires two additional plugins for mathml and svg. You will be prompted to install these plugins when you first login to *WebClass*. Owners of Macintosh computers running OSX should use the Firefox 2.0 web browser available at Firefox **http://www.getfirefox.com**. However, some homework problems will not display correctly without installing additional fonts. Instructions for installing these fonts are given at the FAQ available under the *General* category on the *WebClass* main menu. Students with a Macintosh should consider printing their homework from a Windows computer (this may be done in The Study or Mathskeller). Submission of homework on a Macintosh should not present any difficulty. **Be sure you have Cookies and Popups enabled on your web browser.** This is usually done in the Edit - Preferences menus.

2 Login

The are two methods for login to *WebClass*.

2.1 Active Directory Login

The preferred method for logins to *WebClass* is with the UK Active Directory user name and password. This is also the user name and password that are used to access other systems including *myuk.uky.edu* and *exchange.uky.edu*. Thus, if your user name is *skova01* you will enter *ad\skova01* as the user name and then the password for your Active Directory account. Note that *WebClass* will require you to include the prefix *\ad* while other web sites on campus may not.

2.2 E-Mail and WebClass Password

Students may also log in to their *WebClass* account using their e-mail address and a password that is local to *WebClass*. You may look up the e-mail address for your account by visiting www.mathclass.org, following the link to *login* and then the link *Don't know which User Name or e-mail to use?*. This link will also allow you to look up your Active Directory user name. The initial password will be *u$654321* where *654321* are the last six digits of the student identification number. This is the number on the new student identification card, not the social security number.

2.3 Warning

You may have a different password for each login method. However, both methods give you access to the same account.

3 Account Maintenance

There are two account maintenance chores that each student should perform.

1. Please make sure that you have set up the security questions needed for password resets of your Active Directory account. If you have not done this, visit the account manager at https://ukam.uky.edu/manager.
2. Even if you do not plan to use the e-mail address/mathclass.org password, please log in and change the initial password and security question. Otherwise, a malicious user could obtain access to your account. To change the password for your e-mail address, you must log in with your e-mail address and visit the link *Maintain Your Account* in the yellow box at the top of the page. If you login with your *Active Directory* user name and change the password, you will change your *Active Diretory* password.

4 Problems

1. If you cannot log in, you should check if your *Active Directory* user name and password are correct by logging into another system which uses these passwords. You should also check that you have Cookies enabled in the web browser. If you do not have Cookies enabled you will not be able to login. For Firefox, you enable Cookies in the Edit - Preferences - Privacy menu.
2. If you make five failed login attempts in a thirty minute period, then you may be locked out of your account for thirty minutes.
3. During drop-add, accounts will be created on the next school day after you register for a class. Thus, if you added the class recently, your account may not be created yet. If you change sections, your account and record of homework will be moved automatically.
4. Users may reset the password for their *Active Directory* account by visiting

https://ukam.uky.edu/manager. For additional help with *Active Directory* accounts, call the IT customer service center at 859 257-1300 or visit 111 McVey Hall.

5. If a student resets the *Active Directory* password, the password is reset to the initial password *u$654321* where *654321* stand for the last six digits of the student identification number. You will need to change this initial password by visiting the account manager web site before returning to WebClass. Students may need to wait about 20 minutes before trying to change this password.
6. For technical problems with *WebClass*, visit the link labeled *Help* at the top of most pages of *WebClass*. You do not need to be logged in to visit help.
7. For problems with mathematics, speak with your instructor, teaching assistant or visit Mathskeller or the Study.
8. If you feel a problem was graded incorrectly, speak with your instructor or teaching assistant.
9. Links on the left-hand side of the *WebClass* home page will take you to guides for using *WebClass*. There is an FAQ (Frequently Asked Questions) under the General section of the main floating menu.
10. If you have trouble displaying the *WebClass* pages, be sure you have Cookies and JavaScript enabled on your browser.

Exams

Each exam is worth 100 points. You must bring a photo ID to each exam and you may use a calculator on the exams. Absolutely no cell phone use during an exam is allowed. The final exam, Exam 4, will be comprehensive.

Exam rooms

Click **here** for the room schedule for exams 1-3 and the final exam.

Exam dates and material

Exam 1: 5:00-7:00 pm, Wednesday, February 10, 20xx.
Covers homework sets 00-05 and Chapters 1, 2 and 3.
Exam 2: 5:00-7:00 pm, Wednesday, March 10, 20xx.
Covers homework sets 06 - 12 and Chapters 4, 5, Supplement, and 6 (without concavity).
Exam 3: 5:00-7:00 pm, Wednesday, April 14, 20xx.
Covers homework sets 13-19 and Chapters 6 (concavity), 7, 8 and 9.
Exam 4: 6:00-8:00 pm, Thursday, May 6, 20xx.
Comprehensive (covers all homework sets).

After an exam is given, you should go back over the exam and redo problems you got wrong since this will help you prepare for the final.

Alternate exams

Students who have university excused absences or who have university-scheduled class conflicts with uniform examinations may take the Alternate Exam. You must fill out the Alternate Exam Request Form and submit it to your instructor at least two weeks before the scheduled exam.
Click **here** to download the Alternate Exam Form.
The Alternate Exams for Exams 1-3 are given on the same days as the regular exams (February 10, March 10, April 14) from 7:30 pm - 9:30 pm.

Old exams

Exams from previous semesters are posted **here**.

Old Exams

Exams from previous semesters can be found here.

CAUTION: This semester the exams will differ a little from the old exams. Some new material has been added and some types of questions might be deleted. Nevertheless, the old exams still serve as a rough guide to the material.

Fall 2009	Exam 1 Answer Key	Exam 2 Answer Key	Exam 3 Answer Key	Final Exam Answer Key
Spring 2009	Exam 1 Answer Key	Exam 2 Answer Key	Exam 3 Answer Key	Final Exam Answer Key
Fall 2008	Exam 1 Answer Key	Exam 2 Answer Key	Exam 3 Answer Key	Final Exam Answer Key
Spring 2008	Exam 1 Answer Key	Exam 2 Answer Key	Exam 3 Answer Key	Final Exam Answer Key
Fall 2007	Exam 1 Answer Key	Exam 2 Answer Key	Exam 3 Answer Key	Final Exam Answer Key
Spring 2007	Exam 1 Answer Key	Exam 2 Answer Key	Exam 3 Answer Key	Final Exam Answer Key
Fall 2006	Exam 1 Answer Key	Exam 2 Answer Key	Exam 3 Answer Key	Final Exam Answer Key

Tutoring Resources

If you find that you are having difficulty with any aspect of the course, you should seek help immediately.

If you are having trouble with a homework problem, you can send e-mail through the online homework system to an instructor. Try to provide as much information as possible in your help request. For example, you should at least describe how you attempted the problem and at least guess where you might be going wrong. DO NOT EXPECT to receive e-mail help if you wait until the evening of the due date of the homework assignment.

The Mathematics department will have instructors available in **The Mathskeller**, Whitehall Classroom Building (CB) 63 during the day to answer students' questions between classes M-F from 9am-5pm.

To complement **The Mathskeller** resource, **The Study**, now located on the spacious 3rd Floor Commons on South Campus, will continue to offer free, drop-in Peer Tutoring in the evenings. Peer Tutors for math courses will be available every hour Peer Tutoring is offered: Sundays 6-10pm and M-Th 3-10pm.

Sample Lecture

Chapter Goals:

- Apply the Extreme Value Theorem to find the global maximum and minimum values of a continuous function on a closed and bounded interval.
- Understand the connection between critical points and local extreme values.
- Understand the relationship between the sign of the derivative and the intervals on which a function is increasing and on which it is decreasing.
- Understand the statement and consequences of the Mean Value Theorem.
- Understand how the derivative can help you sketch the graph of a function.
- Understand how to use the derivative to find the global extreme values (if any) of a continuous function over an unbounded interval.
- Understand the connection between the sign of the second derivative of a function and the concavities of the graph of the function.
- Understand the meaning of inflection points and how to locate them.

Assignments: Assignment 12 Assignment 13 Assignment 14

Finding the largest profit, or the smallest possible cost, or the shortest possible time for performing a given procedure or task, or figuring out how to perform a task most productively under a given budget and time schedule are some examples of practical real-world applications of Calculus. The basic mathematical question underlying such applied problems is how to find (if they exist) the largest or smallest values of a given function on a given interval. This procedure depends on the nature of the interval.

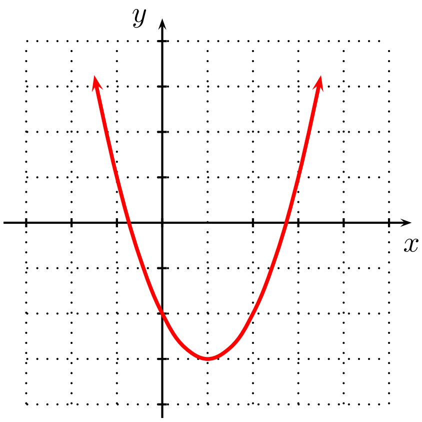
► **Global (or absolute) extreme values:** The largest value a function (possibly) attains on an interval is called its **global (or absolute) maximum value**. The smallest value a function (possibly) attains on an interval is called its **global (or absolute) minimum value**. Both maximum and minimum values (if they exist) are called **global (or absolute) extreme values**.

Example 1:

Find the maximum and minimum values for the function

$$f(x) = (x-1)^2 - 3,$$

if they exist.

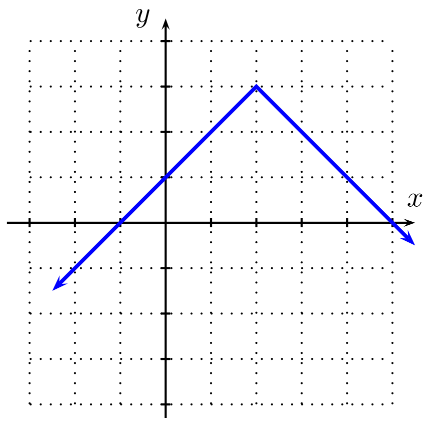


Example 2:

Find the maximum and minimum values for the function

$$f(x) = -|x-2| + 3,$$

if they exist.

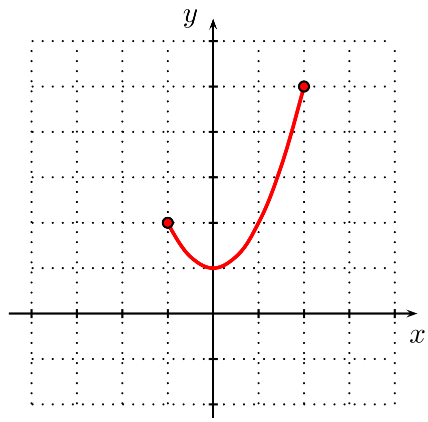


Example 3:

Find the maximum and minimum values for the function

$$f(x) = x^2 + 1, \qquad x \in [-1, 2]$$

if they exist.



We first focus on continuous functions on a closed and bounded interval. The question of largest and smallest values of a continuous function f on an interval that is not closed and bounded requires us to pay more attention to the behavior of the graph of f, and specifically to where the graph is rising and where it is falling.

Closed and bounded intervals:

An interval is **closed and bounded** if it has finite length and contains its endpoints.

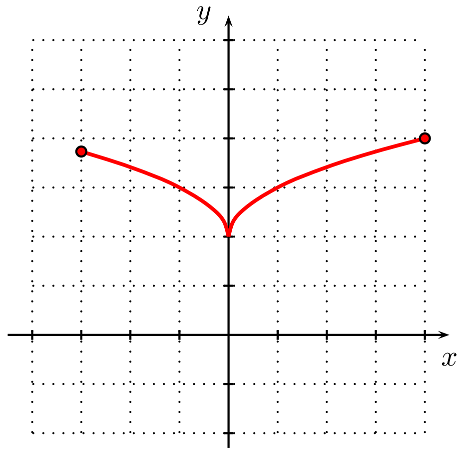
For example, the interval $[-2, 5]$ is closed and bounded.

▶ **The Extreme Value Theorem (EVT):**

If a function f is continuous on a closed, bounded interval $[a, b]$, then the function f attains a maximum and a minimum value on $[a, b]$.

Example 4: Let $f(x) = \begin{cases} 2+\sqrt{x} & \text{if } x > 0 \\ 2+\sqrt{-x} & \text{if } x \leq 0. \end{cases}$

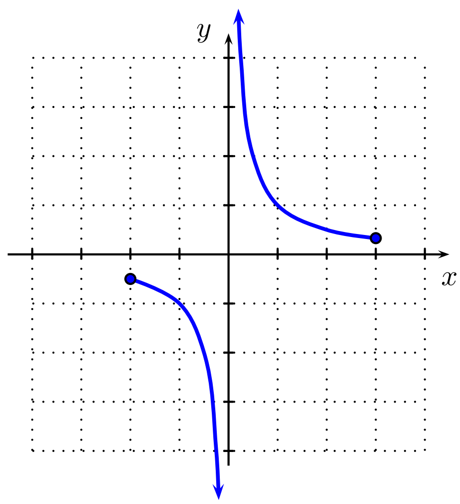
Does $f(x)$ have a maximum and a minimum value on $[-3, 4]$? How does this example illustrate the Extreme Value Theorem?



The following two examples show that the assumptions on f and the interval $[a, b]$ are essential ingredients in the statement of the Extreme Value Theorem.

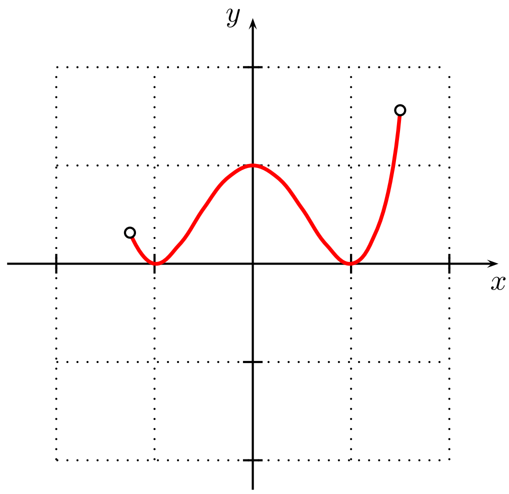
Example 5:

Let $g(x) = \dfrac{1}{x}$. Does $g(x)$ have a maximum value and a minimum value on $[-2, 3]$? Does this example contradict the Extreme Value Theorem? Why or why not?



Example 6:

Let $h(x) = x^4 - 2x^2 + 1$. Does $h(x)$ have a maximum value and a minimum value on $(-1.25, 1.5)$? Does this example contradict the Extreme Value Theorem? Why or why not?



The EVT is an existence statement; it doesn't tell you how to locate the maximum and minimum values of f.

The following results tell you how to narrow down the list of possible points on the given interval where the function f *might* have an extreme value to (usually) just a few possibilities. You can then evaluate f at these few possibilities, and pick out the smallest and largest value.

▶ **Fermat's Theorem:** Let $f(x)$ be a continuous function on the interval $[a, b]$. If f has an extreme value at a point c strictly between a and b, and if f is differentiable at $x = c$, then $f'(c) = 0$.

▶ **Corollary:** Let $f(x)$ be a continuous function on the closed, bounded interval $[a, b]$. If f has an extreme value at $x = c$ in the interval, then either

- $c = a$ or $c = b$;
- $a < c < b$ and $f'(c) = 0$;
- $a < c < b$ and f is not differentiable at $x = c$, so that f' is not defined at $x = c$.

Example 7: Find the maximum and minimum values of $f(x) = x^3 - 3x^2 - 9x + 5$ on the interval $[0, 4]$. For which values x are the maximum and minimum values attained?

Example 8: Find the maximum and minimum values of $F(s) = \dfrac{2s+1}{s-6}$ on the interval $[-1, 5]$. For which values s are the maximum and minimum values attained?

Example 9: Find the maximum and minimum values of $f(x) = x^{2/3}$ on the interval $[-1, 8]$. For which values s are the maximum and minimum values attained?

Example 10: Find the t values on the interval $[-10, 10]$ where $g(t) = |t-4| + 7$ takes its maximum and minimum values. What are the maximum and minimum values?

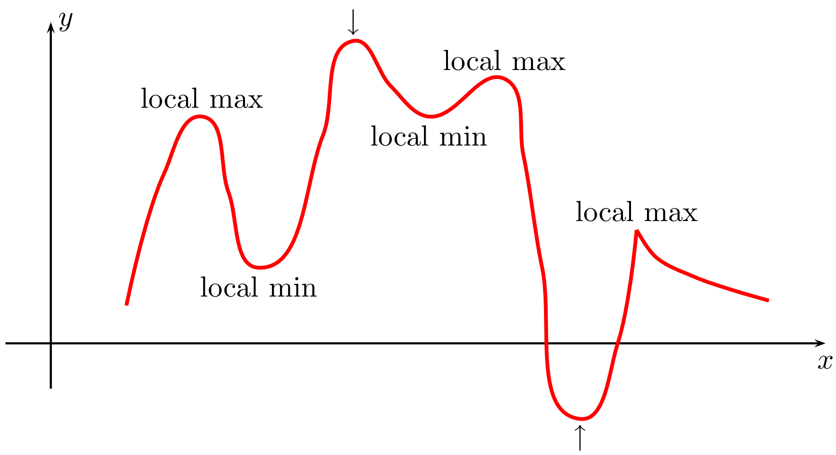
Example 11:

Find the maximum and minimum values of $k(x) = \begin{cases} x^2 + 2x + 1 & \text{if } x \leq 1 \\ -3x + 7 & \text{if } x > 1 \end{cases}$ on the interval $[-2, 3]$.

Example 12: Find the maximum and minimum values of $h(s) = -s - 2s^2 - 3s^3 - 4s^4$ on the interval $[0, 2]$. For which values s are the maximum and minimum values attained?

Example 13: Find the maximum and minimum values of $g(x) = 1 + x + x^2 + x^3$ on the interval $[0, 2]$. For which values x are the maximum and minimum values attained?

▶ **Local (or relative) extreme points:** In addition to the points where a function might have a maximum or minimum value, there are other points that are important for the behavior of the function and the shape of its graph.



If you thought of the graph of the function as the profile of a landscape, the global maximum could represent the highest hill in the landscape, while the minimum could represent the deepest valley. The other points indicated in the graph, which look like tops of hills (although not the highest hills) and bottom of valleys (although not the deepest valleys), are called **local** (or **relative**) **extreme values**. More precisely,

Definition: A function f has a **local** (or **relative**) **maximum** at a point $(c, f(c))$ if there is some interval about c such that $f(c) \geq f(x)$ for all x in that interval. A function f has a **local** (or **relative**) **minimum** at a point $(c, f(c))$ if there is some interval about c such that $f(c) \leq f(x)$ for all x in that interval.

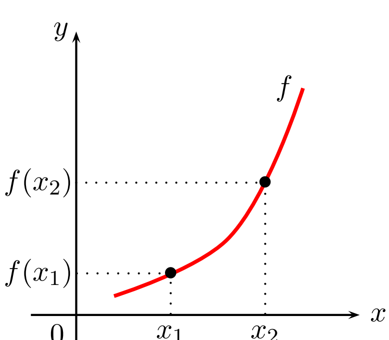
Theorem: If f has a local extreme value at $(c, f(c))$ and is differentiable at that point c, then $f'(c) = 0$

Critical points: Let f be a function. If f is defined at the point $x = c$ and either $f'(c) = 0$ or $f'(c)$ is undefined then the point c is called a **critical point** of f.

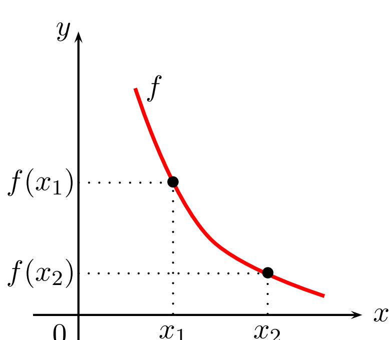
▶ **Increasing and decreasing functions:** A function f is said to be increasing when its graph rises and decreasing when its graph falls. More precisely, we say that:

f is **increasing** on an interval I if
$f(x_1) < f(x_2)$ whenever $x_1 < x_2$ in I.

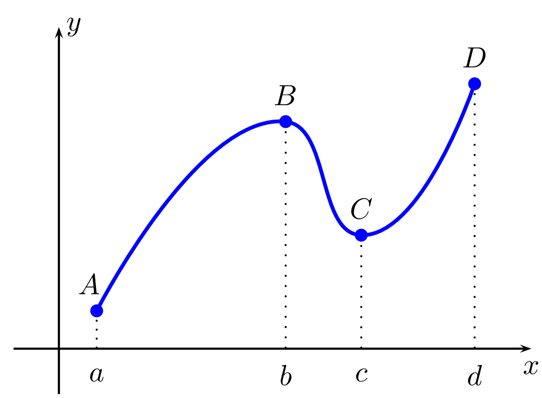
f is **decreasing** on an interval I if
$f(x_1) > f(x_2)$ whenever $x_1 < x_2$ in I.



f is increasing



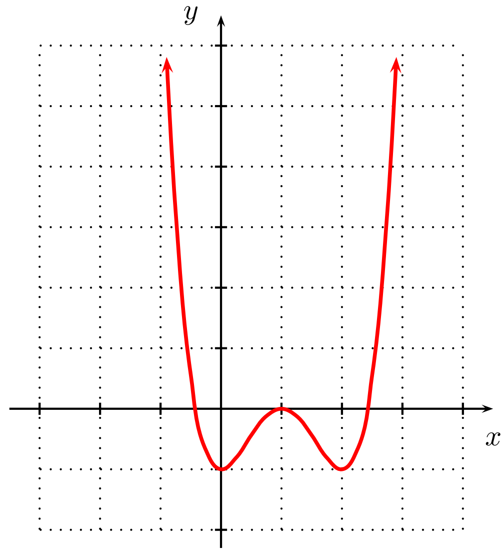
f is decreasing



f is increasing on the intervals $[a, b]$ and $[c, d]$
f is decreasing on the interval $[b, c]$.

Example 14:

Find the intervals over which the function in the graph is increasing and decreasing.

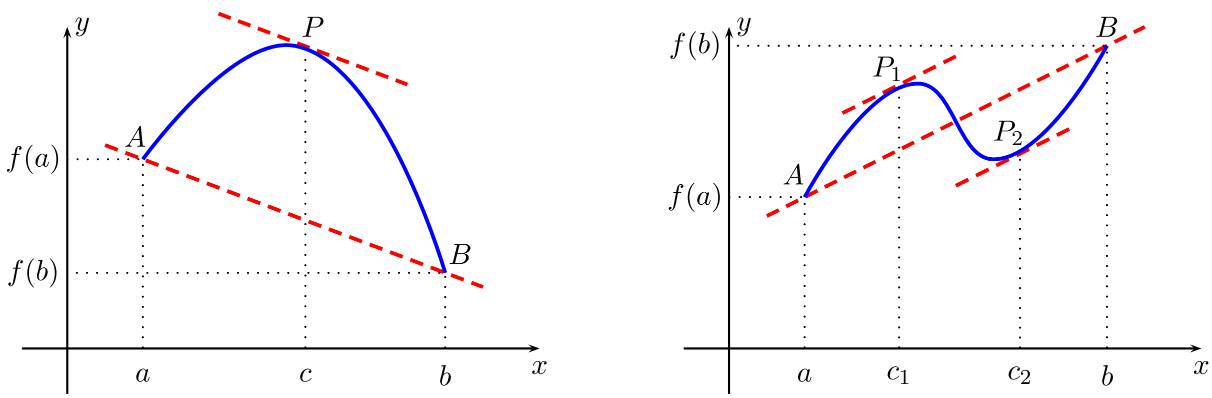


▶ **The Mean Value Theorem (MVT):**

If f is continuous on $[a, b]$ and differentiable at every point strictly between a and b, then there exists some point $x = c$ (and maybe more than one) strictly between a and b such that

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

Geometric interpretation of the MVT: For some (non-necessarily unique) point c between a and b the tangent line to the graph of f at $P(c, f(c))$ has the same slope as the secant line connecting the points $A(a, f(a))$ and $B(b, f(b))$ on the graph of f.



A rewording of the MVT:

If f is continuous on $[a, b]$ and differentiable at every point strictly between a and b, then there exists some point $x = c$ (and maybe more than one) strictly between a and b such that

$$\begin{matrix}\text{the average rate of change} \\ \text{of } f \text{ on } [a,b]\end{matrix} = \begin{matrix}\text{the instantaneous rate of change} \\ \text{of } f \text{ at } x = c.\end{matrix}$$

Note: Example 16 in Chapter 2 is an illustration of the MVT. Two additional examples are proposed next.

Example 15: Let $Q(t) = t^2$. Find a value $A \neq 1$ such that the average rate of change of $Q(t)$ from 1 to A equals the instantaneous rate of change of $Q(t)$ at $t = 2$.

Example 16: Let $f(x) = x - x^3$. Verify that the function satisfies the hypotheses of the Mean Value Theorem on the interval $[-2, 0]$ Then find all numbers c that satisfy the conclusion of the Mean value Theorem.

Here are three consequences of the Mean Value Theorem:

Theorem:

If f is differentiable on an interval I and the derivative $f'(x) = 0$ for all $x \in I$, then f is constant on I.

Corollary:

If f and g are differentiable on an interval I and $f'(x) = g'(x)$ for all $x \in I$, then $f - g$ is constant on I; that is, $f(x) = g(x) + c$ where c is a constant.

Theorem (First derivative test for increasing and decreasing functions):

If f is differentiable on an interval I and $f'(x) > 0$ for all points $x \in I$, then f is increasing on I.

If f is differentiable on an interval I and $f'(x) < 0$ for all points $x \in I$, then f is decreasing on I.

Example 17: Let $f(x) = \dfrac{x+4}{x+7}$. Find the intervals over which the function is increasing.

Example 18: Find the smallest value of A such that the function $f(t) = t^4 - 10t^2 + 9$ is increasing for all t in the interval $(A, 0)$.

Example 19: Find the largest value of A such that the function $h(s) = \dfrac{1}{(s-9)^4}$ is increasing for all s in the interval $(-\infty, A)$.

First derivative test for (local) maxima and minima: If f has a critical value at $x = c$, then

- f has a local maximum at $x = c$ if the sign of f' around c is $\underset{c}{+ + + \,|\, - - - - -}$
- f has a local minimum at $x = c$ if the sign of f' around c is $\underset{c}{- - - - - \,|\, + + +}$

Example 20: Find the local and global extrema, if any, of $f(x) = x^2 e^{-x}$ for $-\infty < x < \infty$.

Example 21: Suppose $k'(t) = (t-5)(t+1)(t-3)$. Find the largest value of A such that the function $k(t)$ is decreasing for all t in the interval $(3, A)$.

Example 22: Suppose $u'(x) = (x^2+1)(x-3)(x-1)(x+5)$. Find the x value in the interval $[-5, 3]$ where $u(x)$ takes its maximum value.

Example 23:

Suppose $g'(x) = 1 + x^2 + x^4$. Find the x values in the interval $[-3, 4]$ where $g(x)$ takes its minimum.

Example 24:

Suppose $g(x) = \dfrac{\sqrt{x-3}}{x}$. Find the value of x in the interval $[3, +\infty)$ where $g(x)$ takes its maximum.

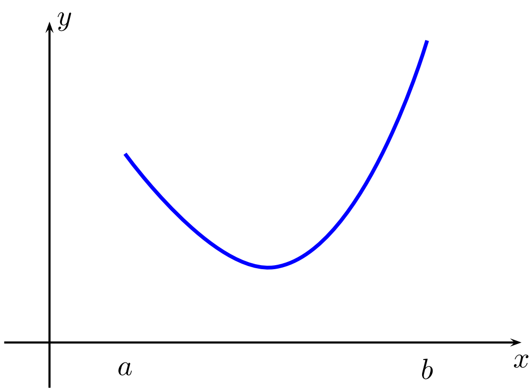
▶ **Curve sketching:** Information on the first derivative can be used to help us sketch the graph of a function. For example, the first derivative can be used to determine where a function is increasing and where it is decreasing.

Example 25: Find the intervals where the function $f(x) = x^3 - 3x^2 + 1$ is increasing and the ones where it is decreasing. Use this information to sketch the graph of $f(x) = x^3 - 3x^2 + 1$.

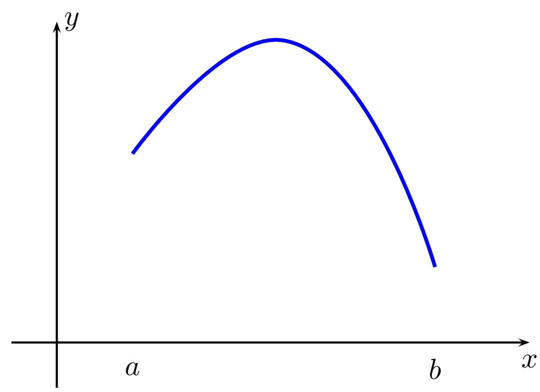
▶ **Concavity:**

The second derivative can also be used to help sketch the graph of a function. We will see that the second derivative can be used to determine when the graph of a function is concave upward or concave downward.

The graph of a function $y = f(x)$ is **concave upward** on an interval $[a, b]$ if the graph lies above each of the tangent lines at every point in the interval $[a, b]$. The graph of a function $y = f(x)$ is **concave downward** on an interval $[a, b]$ if the graph lies below each of the tangent lines at every point in the interval $[a, b]$.



graph of function concave upward on $[a, b]$



graph of function concave downward on $[a, b]$

Second derivative test for concavity: Consider a function $f(x)$.

If $f''(x) > 0$ over an interval $[a, b]$, then the derivative $f'(x)$ is increasing on the interval $[a, b]$. That means the slopes of the tangent lines to the graph of $y = f(x)$ are increasing on the interval $[a, b]$. From this it can be seen that the graph of the function $y = f(x)$ is concave upward.

If $f''(x) < 0$ over an interval $[a, b]$. Then the derivative $f'(x)$ is decreasing on the interval $[a, b]$. That means the slopes of the tangent lines to the graph of $y = f(x)$ are decreasing on the interval $[a, b]$. From this it can be seen that the graph of the function $y = f(x)$ is concave downward.

Inflection points: A point $(c, f(c))$ on the graph is called a **point of inflection** if the graph of $y = f(x)$ changes concavity at $x = c$. That is, if the graph goes from concave up to concave down, or from concave down to concave up. If $(c, f(c))$ is a point of inflection on the graph of $y = f(x)$ and if the second derivative is defined at this point, then $f''(c) = 0$.

Thus, points of inflection on the graph of $y = f(x)$ are found where either $f''(x) = 0$ or the second derivative is not defined. However, if either $f''(x) = 0$ or the second derivative is not defined at a point, it is not necessarily the case that the point is a point of inflection. Care must be taken. In this course however, points of inflection will tend to occur precisely at those points where $f''(x) = 0$.

Example 26: Find the intervals over which the function $f(x) = x^4 - 6x^3 + 12x^2 + 3x - 1$ is concave upward and the ones over which it is concave downward. Locate the x-coordinate of the inflection points of f.

Example 27: If the derivative of the function $g(x)$ is given by $g'(x) = 4x^2 + 12x + 15$, determine the interval(s) where $g(x)$ is concave upward and the one(s) where it is concave downward. Find the x-coordinate of the inflection point(s).

Example 28: Find the x-coordinate of the inflection points of the function $g(x) = e^{-x^2}$.

Example 29:

Let $h(x) = (x+3)\ln(x-5)$ for $x > 5$. Find the interval over which $h(x)$ is concave upward.

Example 30: Let $h(x) = xe^{-x}$. Find the interval over which $h(x)$ is concave downward.

Example 31: Find the coordinates of the inflection point of the function $g(x) = x\sqrt{x^2+3}+4$.

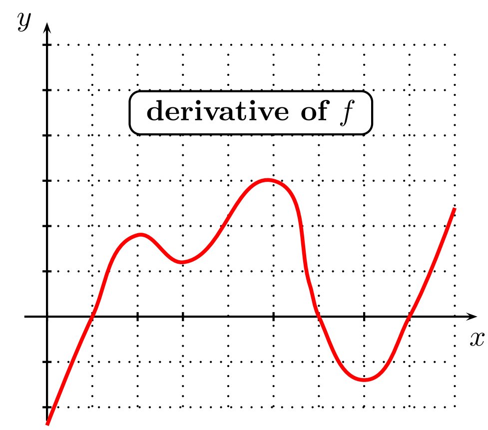
Example 32: The graph of the <u>derivative</u> f' of a function f is shown.

(a) On what intervals is f increasing or decreasing?

(b) At what values of x does f have a local maximum or minimum?

(c) On what intervals is f concave upward or downward?

(d) State the x-coordinate of the inflection points of f.



Spreadsheet Project

Tutorial

Example 1:

Guess the limit or state that it does not exist.
lim (3x^2+5)
x->2

Solution:

We need to evaluate 3x^2+5 at x values that are close to 2. This can be computationally tedious, but computers can help. See the chart below.

x	3x^2+5
1.9	15.83
1.99	16.8803
1.999	16.988
1.9999	16.9988
1.99999	16.99988
2.00001	17.00012
2.0001	17.0012
2.001	17.012
2.01	17.1203
2.1	18.23

Click on the cells to see how the formulas are entered. The formulas will appear in the white box at the top of the page. Notice that each formula begins with an equal sign.

Based on the data above, it is reasonable to guess that
lim (3x^2+5) = 17
x->2

Life Made Easier with Copy and Paste:

At this point, it might seem that it would be just as easy to use a calculator as it is to use a spreadsheet. Entering all those formulas would be very time-consuming. In fact, you only need to enter the formula one time. Copy and paste can take care of the rest. Let's rework the previous example. You are given the first column of the chart. Follow the instructions below to complete the chart.

x	3x^2+5
1.9	
1.99	
1.999	
1.9999	
1.99999	
2.00001	
2.0001	
2.001	
2.01	
2.1	

1. Click on the green cell.
2. In the green cell, type:
=3*b44^2+5
Hit <ENTER>.
3. Right click on the green cell.
4. Click on "Copy."
5. Use your mouse to highlight the rest of the cells in the rightmost column.
6. Right click in the highlighted area.
7. Click Paste.
8. Does your chart match the chart in the previous example? Click on some of the cells in the rightmost column of the chart to see the formulas used to calculate the value in the cell.

Example 2:

Guess the limit or state that it does not exist.
lim x^2-2x-15
x->5 x-5

$$\lim_{x \to 5} \frac{x^2-2x-15}{x-5}$$

x	(x^2-2x-15)/(x-5)
4.9	
4.99	
4.999	
4.9999	
4.99999	
5.00001	
5.0001	
5.001	
5.01	
5.1	

Your Turn!

1. Click on cell B62.
2. Enter the correct formula for this new function. Be sure to start the formula with an equal sign. You should also put parentheses around the numerator and parentheses around the denominator.
3. Hit Enter.
4. Copy and Paste your formula.
5. Use your chart to help you answer the question.
6. Check your answer by clicking on the Answers tab of this worksheet. (Tabs are at the bottom of the page.)

Example 3:

Guess the limit or state that it does not exist.

$$\lim_{x \to 1} \frac{x^2+4}{x-1}$$

x	(x^2+4)/(x-1)

Your Turn!

1. Try this on your own. You will need to enter x values and a formula.
2. Check your answer by clicking on the Answers tab of this worksheet. (Tabs are at the bottom of the page.)

Example 4:

Let f(x)=3x^2-1. Guess f'(3) or state that it does not exist.

Recall that :
f'(3) = lim $\lim_{h \to 0} \frac{f(3+h)-f(3)}{h}$
h->0

Because h is approaching zero, the first column of our chart should contain h values that are close to zero. Remember that we should some h values that are bigger than zero and some that are smaller than zero. It is possible to solve this problem using a chart that contains only two columns. However, we are going to make a chart with 4 columns because it simplifies the formulas.

h	f(3+h)	f(3)	(f(3+h)-f(3))/h
-0.1	24.23	26	17.7
-0.01	25.8203	26	17.97
-0.001	25.982	26	17.997
-0.0001	25.9982	26	17.9997
-0.00001	25.99982	26	17.99997
0.00001	26.00018	26	18.00003
0.0001	26.0018	26	18.0003
0.001	26.018	26	18.003
0.01	26.1803	26	18.03
0.1	27.83	26	18.3

1. Pay attention to the formulas in these boxes.

2. We only entered values for h and formulas for the first row of the chart. All other values were obtained by using copy and paste.

Based on the data above, it is reasonable to guess that f'(3)=18.

Example 5:

Let f(x)=x^3+1. Guess f'(2) or state that it does not exist.

h	f(2+h)	f(2)	(f(2+h)-f(2))/h
-0.1			
-0.01			
-0.001			
-0.0001			
-0.00001			
0.00001			
0.0001			
0.001			
0.01			
0.1			

Your Turn!

1. Try this on your own. You will need to enter the appropriate formulas.

2. Check your answer by clicking on the Answers tab of this worksheet. (Tabs are at the bottom of the page.)

Example 6:

Let $f(x)=\frac{x+1}{x-2}$. Guess f'(4) or state that it does not exist.

h			

Your Turn!

1. Try this on your own. You will need to enter values for h, the appropriate column headings, the appropriate formulas.

2. Check your answer by clicking on the Answers tab of this worksheet. (Tabs are at the bottom of the page.)

Answers to Tutorial

Example 1:

Guess the limit or state that it does not exist.
lim (3x^2+5)
x->2

Solution:

We need to evaluate 3x^2+5 at x values that are close to 2. This can be computationally tedious, but computers can help. See the chart below.

x	3x^2+5
1.9	15.83
1.99	16.8803
1.999	16.988
1.9999	16.9988
1.99999	16.99988
2.00001	17.00012
2.0001	17.0012
2.001	17.012
2.01	17.1203
2.1	18.23

Click on the cells to see how the formulas are entered. The formulas will appear in the white box at the top of the page. Notice that each formula begins with an equal sign.

Based on the data above, it is reasonable to guess that
lim (3x^2+5) = 17
x->2

Life Made Easier with Copy and Paste:

At this point, it might seem that it would be just as easy to use a calculator as it is to use a spreadsheet. Entering all those formulas would be very time-consuming. In fact, you only need to enter the formula one time. Copy and paste can take care of the rest. Let's rework the previous example. You are given the first column of the chart. Follow the instructions below to complete the chart.

x	3x^2+5
1.9	16.8803
1.99	16.988
1.999	16.9988
1.9999	16.99988
1.99999	17.00012
2.00001	17.0012
2.0001	17.012
2.001	17.1203
2.01	18.23
2.1	5

1. Click on the green cell.
2. In the green cell, type:
=3*b44^2+5
Hit <ENTER>.
3. Right click on the green cell.
4. Click on "Copy."
5. Use your mouse to highlight the rest of the cells in the rightmost column.
6. Right click in the highlighted area.
7. Click Paste.
8. Does your chart match the chart in the previous example? Click on some of the cells in the rightmost column of the chart to see the formulas used to calculate the value in the cell.

Example 2:

Find the limit or state that it does not exist.
lim   x^2-2x-15
x->5   x-5

x	(x^2-2x-15)/(x-5)
4.9	7.9
4.99	7.99
4.999	7.999
4.9999	7.9999
4.99999	7.99999
5.00001	8.00001
5.0001	8.0001
5.001	8.001
5.01	8.01
5.1	8.1

Your Turn!

1. Click on cell B62.
2. Enter the correct formula for this new function. Be sure to start the formula with an equal sign. You should also put parentheses around the numerator and parentheses around the denominator.
3. Hit Enter.
4. Copy and Paste your formula.
5. Use your chart to help you answer the question.
6. Check your answer by clicking on the Answers tab of this worksheet. (Tabs are at the bottom of the page.)

ANSWER: Based on the data above, appears that the limit equals 8.

Example 3:

Find the limit or state that it does not exist.
lim   x^2+4
x->1   x-1

x	(x^2+4)/(x-1)
0.9	-48.1
0.99	-498.01
0.999	-4998.001
0.9999	-49998.0001
0.99999	-499998
1.00001	500002
1.0001	50002.0001
1.001	5002.001
1.01	502.01
1.1	52.1

Your Turn!

1. Try this on your own. You will need to enter x values and a formula.
2. Check your answer by clicking on the Answers tab of this worksheet. (Tabs are at the bottom of the page.)

ANSWER: Based on this data, it appears that the limit does not exist.

Example 4:

Let f(x)=3x^2-1. Find f'(3) or state that it does not exist.

Recall that :
f'(3) = lim f(3+h)-f(3)
h->0 h

Because h is approaching zero, the first column of our chart should contain h values that are close to zero. Remember that we should some h values that are bigger than zero and some that are smaller than zero. It is possible to solve this problem using a chart that contains only two columns. However, we are going to make a chart with 4 columns because it simplifies the formulas.

h	f(3+h)	f(3)	(f(3+h)-f(3))/h
-0.1	24.23	26	17.7
-0.01	25.8203	26	17.97
-0.001	25.982	26	17.997
-0.0001	25.9982	26	17.9997
-0.00001	25.99982	26	17.99997
0.00001	26.00018	26	18.00003
0.0001	26.0018	26	18.0003
0.001	26.018	26	18.003
0.01	26.1803	26	18.03
0.1	27.83	26	18.3

1. Pay attention to the formulas in these boxes.

2. We only entered values for h and formulas for the first row of the chart. All other values were obtained by using copy and paste.

Based on the data above, it is reasonable to guess that f'(3)=18.

Example 5:

Let f(x)=x^3+1. Find f'(2) or state that it does not exist.

h	f(2+h)	f(2)	(f(2+h)-f(2))/h
-0.1	7.859	9	11.41
-0.01	8.880599	9	11.9401
-0.001	8.988006	9	11.994001
-0.0001	8.9988	9	11.99940001
-0.00001	8.99988	9	11.99994
0.00001	9.00012	9	12.00006
0.0001	9.0012	9	12.00060001
0.001	9.012006	9	12.006001
0.01	9.120601	9	12.0601
0.1	10.261	9	12.61

Your Turn!

1. Try this on your own. You will need to enter the appropriate formulas.

2. Check your answer by clicking on the Answers tab of this worksheet. (Tabs are at the bottom of the page.)

ANSWER: Based on this data, it appears that f'(2)=12 f'(2)=12.

Example 6:

Let $f(x)=\frac{x+1}{x-2}$. Find f'(4) or state that it does not exist.

h	f(4+h)	f(4)	(f(4+h)-f(4))/h
-0.1	2.578947	2.5	-0.789473684
-0.01	2.507538	2.5	-0.753768844
-0.001	2.50075	2.5	-0.750375188
-0.0001	2.500075	2.5	-0.750037502
-0.00001	2.500008	2.5	-0.75000375
0.00001	2.499993	2.5	-0.74999625
0.0001	2.499925	2.5	-0.749962502
0.001	2.49925	2.5	-0.749625187
0.01	2.492537	2.5	-0.746268657
0.1	2.428571	2.5	-0.714285714

ANSWER: Based on this data, it appears that f'(4)=-0.75.

Your Turn!

1. Try this on your own. You will need to enter values for h, the appropriate column headings, the appropriate formulas.

2. Check your answer by clicking on the Answers tab of this worksheet. (Tabs are at the bottom of the page.)

The Project

Question 1:

How do you find the absolute value of a number in Excel?
You may need to do some research to answer this question.

ANSWER:

Question 2:

Complete the chart below. Be sure that all of the columns have labels. Use this information to guess the limit or state that it does not exist.

lim |x-5|
x->5

x	

ANSWER:

Question 3:

Let g(x) = |x-5|. Complete the chart below. Be sure that all of the columns have labels. Use this information to guess g'(2) or state that it does not exist. If g'(2) does not exist, explain why it does not exist.

h			
-0.1			
-0.01			
-0.001			
-0.0001			
-0.00001			
0.00001			
0.0001			
0.001			
0.01			
0.1			

ANSWER:

Question 4:

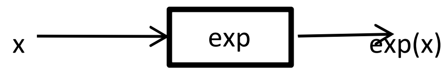
Let g(x) = |x-5|. Complete the chart below. Be sure that all of the columns have labels. Use this information to guess g'(5) or state that it does not exist. If g'(5) does not exist, explain why it does not exist.

h			
-0.1			
-0.01			
-0.001			
-0.0001			
-0.00001			
0.00001			
0.0001			
0.001			
0.01			
0.1			

ANSWER:

Question 5:

Very often we give functions simple names such as f, g, and h. Certain special functions occur so often that they have longer more descriptive names. log, sin, cos, and tan are all names of special functions. One special function, the exponential function, is named exp. The machine diagram for the exp function is shown below.



Excel recognizes the exp function. If you want to know the value of the output of the exponential function when x is 2. you need to type =exp(2) in one of the cells. Complete the chart below to see some of the output values of the exponential function.

x	exp(x)
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	

QUESTION: What formula did you use to calculate the value in cell B127? You should write exactly what you typed in the cell.

ANSWER:

Complete the chart below. Use this information to guess exp'(2) . Compare exp(2) and your guess for exp'(2).

h	exp(2+h)	exp(2)	(exp(2+h)-exp(2))/h
-0.1			
-0.01			
-0.001			
-0.0001			
-0.00001			
0.00001			
0.0001			
0.001			
0.01			
0.1			

ANSWER:

Complete the chart below. Use this information to guess exp'(1) . Compare exp(1) and your guess for exp'(1).

h	exp(1+h)	exp(1)	(exp(1+h)-exp(1))/h
-0.1			
-0.01			
-0.001			
-0.0001			
-0.00001			
0.00001			
0.0001			
0.001			
0.01			
0.1			

ANSWER:

Based on the evidence in this question, make a conjecture about exp'(5). Make a conjecture about exp'(a) for any real number a.

ANSWER:
CONJECTURE: Based on the evidence in this problem, I believe that exp'(5) is equal to ________________________.

CONJECTURE: Based on the evidence in this problem, I believe that exp'(a) is equal to ________________________.

Population Growth Project

A Project on Population Growth
MA 123

NAME: ______________________________

SECTION NUMBER: ______________________________

INSTRUCTIONS: In this project, you will need to show your work. You will not receive credit for unsupported answers.

1. Go to the UK website `http://libguides.uky.edu/content.php?pid=52543&sid=553423`. Read the material at the three links:

   - Evaluating Web Pages: Techniques to Apply and Questions to Ask.
   - Evaluating Internet Information
   - Five Criteria for Evaluating Websites

   and watch the videos:

   - Evaluating Websites
   - Researching Online for College Students

2. Find the world population on July 1, 1965 and July 1, 1970. Round your answer to the nearest tenth of a billion. You will need to some research to answer this question. Be sure to cite the source that you used for your information and write one sentence indicating why your source is credible, based on what you learned in question 1.

3. Using the information you found in question 2, find the average rate of change of the world population with respect to time from July 1, 1965 to July 1, 1970. Include the appropriate units in your answer.

4. In class we have studied the exponential growth model. Assume that the population of the world is growing exponentially. Use the data that you found in question 2 to write a model for the population P of the world t years after July 1, 1965. The units on P should be billions of people.

5. Use the exponential growth model you found in question 4 to estimate the population of the world on July 1, 2009 and on July 1, 2050.

   Find the world population estimates of the U.S. Census Bureau for July 1, 2009 and July 1, 2050. How do these estimates compare with your estimates?

6. Another model for population growth is the logistic model. This model assumes that there is a maximum population, also known as a carrying capacity, and that the rate of population growth slows as the population approaches the carrying capacity. The variables for a logistic model are defined below.

   - t - time, the number of years since July 1, 1965
   - $P(t)$ - the population at time t in billions of people
   - P_0 - the population at time $t = 0$ in billions of people
   - M - the maximum population or carrying capacity in billions of people
   - k - a constant

   The logistic model for population growth is given by:

   $$P(t) = \frac{MP_0}{P_0 + (M - P_0)e^{-kt}}$$

   The human carrying capacity of the earth is a very controversial subject. According to Joel E. Cohen, estimates for the human carrying capacity of the earth have ranged from less than 1 billion to more than 1 trillion people. Cohen states, "Such estimates deserve the same profound skepticism as population projections" [1]. With the understanding that estimates for human carrying capacity warrant skepticism, let us consider the implications of a carrying capacity of 12 billion people. (Cohen calculated the median of 65 upper bounds on human carrying capacity to be 12 billion people [1].)

   Assuming that the human carrying capacity of the earth is 12 billion people, find a logistic model for the world population using the data that you found in question 2.

7. Use the logistic growth model you found in question 6 to estimate the population of the world on July 1, 2009 and on July 1, 2050.

   Compare your estimates to the world population estimates of the U.S. Census Bureau for July 1, 2009 and July 1, 2050.

8. Based on the population data that you have July 1, 2009 and your population estimates for July 1, 2009, does the exponential growth model or the logistic growth model do a better job of estimating population? Briefly justify your answer.

References

[1] J. E. Cohen, *Population growth and earth's human carrying capacity*, Science, 269 (1995), pp. 341–346.