approved by Undergraduate Council

Sharon Gill

Digitally signed by Sharon Gill
DN: cn=Sharon Gill, o=Undergraduate Education, ou=Undergraduate Council, email=sgill@uky.edu, c=US
Date: 2011.01.13 13:56:50 -05'00'

General Education Course Approval Form

Date of Submission: **April 22, 2010**

1. Check which area(s) this course applies to.

Inquiry – Arts & Creativity	☐	Composition & Communications - II	☐
Inquiry – Humanities	☐	Quant Reasoning – Math	x
Inquiry – Nat/Math/Phys Sci	☐	Quant Reasoning – Stat	☐
Inquiry – Social Sciences	☐	Citizenship – USA	☐
Composition & Communications - I	☐	Citizenship - Global	☐

2. Provide Course and Department Information.

Department: Mathematics

Course Prefix and Number: MA 113 Credit hours: 4

Course Title: Calculus I

Expected Number of Students per Section: 25 Course Required for Majors in your Program? Yes

Prerequisite(s) for Course? *Math ACT of 27 or above, or math SAT of 620 or above, or MA 109 and MA 112, or MA 110, or consent of the department.*

Departmental Contact Information Date: ☒ existing course

Name: Zhongwei Shen Email: zshen2@email.uky.edu

Office Address: POT 721 Phone: 257-3470

3. In addition to this form, the following must be submitted for consideration:

- A major course change form for revision of existing courses or a new course form for new courses.
- A syllabus that conforms to the Senate Syllabi Guidelines, including listing of the Course Template Student Learning Outcomes.
- A narrative that explains: 1) how the course will address the General Education and Course Template Learning outcomes; 2) active learning activities for students; and 3) the course assignment(s) that can be used for Gen Ed course assessment.

4. Signatures

Department Chair: [signature] Date: April 22, 2010

Dean: Anna R. K. Bosch [signature] Date: 8/5/10

Submit all proposals electronically to:
Sharon Gill
Office of Undergraduate Education
Sharon.Gill@uky.edu

Revised March 2010

Hanson, Roxie

From: R. Brown [rmb.uky.math@gmail.com]
Sent: Friday, May 14, 2010 5:19 PM
To: Hanson, Roxie
Cc: Shen, Zhongwei (shenz@ms.uky.edu)
Subject: Re: MA 111/113 need narrative for gen ed

Oops, sorry. I misread your message. I thought you were asking about
123 and 113.

MA 111 is already approved.
MA 113 was presented to the committee before the new procedures was invented.

All four of our gened packets are at the website, http://www.math.uky.edu/~rbrown/dus/

Russell Brown

2010/5/14 R. Brown <rmb.uky.math@gmail.com>:
> I think these were sent directly to the committee before the new
> procedure for GenEd course approval was invented.
>
> But if you would like another copy, visit
> http://www.math.uky.edu/~rbrown/dus/
> and scroll down.
>
> Russell Brown
>
>
> 2010/5/14 Hanson, Roxie <rhanson@email.uky.edu>:
>> Professor Shen, I have hard copies of the syllabus and the Gen Ed
>> course approval forms. I am missing the following:
>>
>>
>>
>> ·   A narrative (2-3 pages max) that explains: 1) how the
>> course will address the General Education and Course Template
>> Learning outcomes; and 2) a description of the type(s) of course
>> assignment(s) that could be used for Gen Ed assessment.
>>
>>
>>
>> Best, Roxie
>
>
>
> --
> Russell Brown :-: russell.brown@uky.edu
> ==
> If I were founding a university I would begin with a smoking room;
> next a dormitory; and then a decent reading room and a library. After
> that, if I still had more money that I couldn't use, I would hire a
> professor and get some text books.
> --Stephen Leacock
>

--
Russell Brown :-: russell.brown@uky.edu
==
If I were founding a university I would begin with a smoking room; next a dormitory; and then a decent reading room and a library. After that, if I still had more money that I couldn't use, I would hire a professor and get some text books.
--Stephen Leacock

UK

Department of Mathematics
College of Arts and Sciences
University of Kentucky

Documents related to the Undergraduate program.

GenEdification of MA 137, 12 April 2010

- Packet regarding MA 137 to be submitted to the quantitative foundations vetting committee

Course in game theory, 5 April 2010

- Syllabus
- Rationale for offering course
- New course form

The GenEdification of MA 123, 22 March 2010

- Proposal to include MA 123 as a course to satisfy the quantitative foundations requirement of the new GenEd program

The GenEdification of MA 113, 6 February 2010

- Packet for submission to the quantitative foundations vetting committee

MA 109 prerequisite change, 1 February 2010

- MA 109 prerequisite change

The GenEdification of MA 111, 25 January 2010

- Proposal to add MA 111 to the University's general education program

Course prerequisites, 15 January 2010

- Minor changes to course prerequisites

Mathematical sciences major, to be considered on 29 October 2009

- Mathematical sciences option, BS
- Mathematical sciences option, BA
- Cover memo

Problem solving for middle-school teachers, to be considered on 29 October 2009.

- Form
- Syllabus

University Senate Syllabi Guidelines

MA 113
Gen Ed

General Course Information

- ☑ Full and accurate title of the course.
- ☑ Departmental and college prefix.
- ☑ Course prefix, number and section number.
- ☑ Scheduled meeting day(s), time and place.

Instructor Contact Information (if specific details are unknown, "TBA" is acceptable for one or more fields)

- ☐ Instructor name.
- ☐ Contact information for teaching/graduate assistant, etc.
- ☐ Preferred method for reaching instructor.
- ☐ Office phone number.
- ☐ Office address.
- ☐ UK email address.
- ☐ Times of regularly scheduled office hours and if prior appointment is required.

Course Description

- ☑ Reasonably detailed overview of the course.
- ☑ Student learning outcomes.
- ☑ Course goals/objectives.
- ☑ Required materials (textbook, lab materials, etc.).
- ☑ Outline of the content, which must conform to the Bulletin description.
- ☑ Summary description of the components that contribute to the determination of course grade.
- ☑ Tentative course schedule that clarifies topics, specifies assignment due dates, examination date(s).
- ☑ Final examination information: date, time, duration and location.
- ☑ For 100-, 200-, 300-, 400-, 400G- and 500-level courses, numerical grading scale and relationship to letter grades for *undergraduate* students.
- N/A ☐ For 400G-, 500-, 600- and 700-level courses, numerical grading scale and relationship to letter grades for *graduate* students. (Graduate students cannot receive a "D" grade.)
- ☑ Relative value given to each activity in the calculation of course grades (Midterm=30%; Term Project=20%, etc.).
- ☑ Note that undergraduate students will be provided with a Midterm Evaluation (by the midterm date) of course performance based on criteria in syllabus.
- ☑ Policy on academic accommodations due to disability. Standard language is below:

  If you have a documented disability that requires academic accommodations, please see me as soon as possible during scheduled office hours. In order to receive accommodations in this course, you must provide me with a Letter of Accommodation from the Disability Resource Center (Room 2, Alumni Gym, 257-2754, email address jkarnes@email.uky.edu) for coordination of campus disability services available to students with disabilities.

Course Policies

- ☑ Attendance.
- ☑ Excused absences.
- ☐ Make-up opportunities.
- ☑ Verification of absences.
- ☑ Submission of assignments.
- ☑ Academic integrity, cheating & plagiarism.
- ☑ Classroom behavior, decorum and civility.
- N/A ☐ Professional preparations.
- N/A ☐ Group work & student collaboration.

To: Arny Stromberg, chair quantitative foundations vetting committee
From: Russell Brown, dus math
Subject: Proposal to include MA 113, Calculus I in the new General Education program
Date: February 6, 2010

Attached, find a syllabus and sample assignments for MA 113, Calculus I. The Department of Mathematics would like to request that this course be approved as one path that students may use to satisfy part III a), Quantitative Foundations, of the proposed new General Education program. We expect that this course will be taken by students who are considering majors with high expectations for quantitative skills. Undeclared students and students with an interest in mathematics may choose this course also. The Department intends to offer a variety of courses to allow students to choose a course that is appropriate to their intended course of study and background in mathematics.

We discuss briefly how this course will meet the learning outcomes in the course templates.

Learning outcome 1. Students in MA 113, Calculus I, are asked to write functional models and then use Calculus to answer questions about these models. Typical problems include writing a quadratic function to give the height of an object that is falling under the influence of gravity One might might be asked to find the height a particular time or to find the time when the object is at its highest point. The emphasis is not on memorizing formulae which answer questions, but being able to lay out the reasoning behind these formulae.

Another class of problems arise from geometry where one is asked to find tangent lines to curves with specified properties or to sketch a curve using the information about the derivative. Again, these problems have concrete realizations which can be visualized by drawing graphs on paper or with the aid of devices such as a calculator. In these problems, students are expected to relate algebraic or symbolic information to geometric properties of the graphs.

We believe that this course provides ample preparation in studying deterministic relationships between numerical quantities and will prepare students for the more sophisticated statistical inferential reasoning where relationships are studied with an additional element of uncertainty added.

Learning outcome 2. Students in MA 113, Calculus I are expected, at least at a rudimentary level, to provide correct explanations for the mathematical statements that they make. In recitation classes, students are given the opportunity to make presentations to their peers, on exams students are expected to provide written justification of their work, and students are given a small number of longer written assignments that ask them to work through a several step solution to a problem and provide written justification of their work. Web homework is designed to include a problems which approach an idea from a variety of directions, rather than series of rote exercises. We do not typically ask for in depth explanations on web homework problems as we are not yet ready to automatically grade these responses. Rather, we try to develop good habits by providing students with a variety of problems so that memorizing recipes is not an efficient way of producing correct solutions.

In addition to providing examples of correct reasoning through presenting mathematics, students are asked to study examples that illustrate typical logical fallacies. Thus, a student will see the distinction between a logical implication (if a function is differentiable, then it is continuous) its converse (if a function continuous, then it is differentiaable) by studying examples of functions which are continuous, but not differentiable.

Information literacy. We believe that mathematics is a fundamental skill in processing information and thus the information literacy component of the course will be focused primarily on the deductive reasoning techniques that we are working to develop in our students.

In addition, we present a small number of additional assignments that ask students to find more information about the historical development of the subject of Calculus.

To assess our students learning, we will rely on the longer written assignments. For the past few semesters, students have been asked to complete six written assignments and these are graded by teaching assistants using a uniform rubric developed by the course coordinator. The ideal assignment will ask a student to formulate a more-or-less real life problem in mathematical terms, illustrate this problem by examples, develop a careful mathematical solution of this problem, and interpret the mathematics in terms of the original problem. Several examples of such problems from recent semesters are attached to this proposal. Several of these assignments are locally written (though we are not so bold as to claim that we have invented a new Calculus problem) and others are taken from our current textbook, Calculus (Early Transcendentals), 6th edition by James Stewart. Stewart's textbook provides additional assignments (projects in Stewart's terminology) that will serve to provide variety to our courses in the coming years.

Attached to this memorandum, please find a syllabus and course calendar for MA 113, Calculus I, and sample written assignments. Several of these have been used in past semesters in our courses. A few are intended to address the information literacy component of the new General Education program and have not been used in the past.

Attachments:

- Current syllabus for MA 113.
- Course calendar of assignments
- Project from Stewart: Early methods for finding tangents
- Sample written assignment: Tangent lines to circles

- Sample written assignment: Tangent lines to parabolas
- Sample written assignment: The law of cosines and differentiation rules
- Sample written assignment: Optimal shape of a tin can
- Project from Stewart: The shape of a can
- Sample written assignment: The volume of a pyramid
- Sample written assignment: The calculus of rainbows
- Project from Stewart: The calculus of rainbows
- Project from Stewart: The origin of L'Hospital's rule
- Project from Stewart: Newton, Leibniz, and the invention of Calculus

Syllabus for MA 113 – Calculus I, Spring 2010

Web site:

The home page for this course is at

http://www.ms.uky.edu/~heidegl/Ma113S10/Ma113.html

It is designed to help you and to provide information. This syllabus, the course calendar, all handouts and solutions to exams and written assignments will be posted on this web site.

Class Schedule:

- Lectures: MWF, time and place according to your section (see also web page)
- Recitations: Time and place according to your section (see also web page)
- **Exams**: There are three uniform midterm exams and one final exam. The final exam will be cumulative though with an emphasis on the material covered since the third exam. Exam rooms will be announced later.

  Exam 1: Tue, Feb. 9, 7:30 – 9:30 pm

  Exam 2: Tue, Mar. 9, 7:30 – 9:30 pm

  Exam 3: Tue, Apr. 13, 7:30 – 9:30 pm

  Final exam: Wed, May 5, 6:00 - 8:00 pm

  For alternate exam procedures, see the section on Policies.

Textbook:

Calculus (Early Transcendentals), 6th edition, by James Stewart, ISBN 978-0-495-01166-8 or 0-495-01166-5. The book *Single Variable - Calculus (Early Transcendentals)*, 6th edition, by James Stewart may also be used.

Recitation Worksheets:

These worksheets are mandatory for the course. They can be purchased at the bookstore or downloaded from http://www.math.uky.edu/~ma113/worksheets-s-10.pdf.

MA 193:

In addition to the 4 hours of credit for MA 113, the department offers one additional hour of credit for MA 193 on a pass/fail basis. You will pass MA 193 if you have at most 2 unexcused absences during MA 113 recitations and pass MA 113. If you fail MA 113 or have 3 or more unexcused absences you will fail MA 193. You are responsible for bringing the recitation worksheets to recitation. Failure to do so will result in an unexcused absence.

Your section number for MA 193 has to equal your section number for MA 113. If you drop or change sections of MA 113, please make sure to also drop or change sections of MA 193. **It is your responsibility to take care of this if you change sections; otherwise you risk a failing grade for MA 193 because you are not on the proper class roll.**

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1. Web-based Homework:
The bulk of the homework will be completed using the well tested **web-based homework** system that grades your solutions and records your scores. You find it at `www.mathclass.org` (see below for administrative details on using this website). Each homework set comes as a common version and a personal version. When entering answers to the common version into the system, it will tell you whether or not your answer was correct and, if necessary, provides you with the correct solution. When entering answers to the personal version you will see whether or not it is correct, but nothing else. This way, the common version serves as a study guide for the personal version. **Only correct solutions to your personal version of the homework assignment give you credit!** Notice that for each web-based homework problem **you may resubmit your answer as often as you wish before midnight of the due date!** Only your final (and hopefully correct answer) will be recorded for your homework grade. You may find your score at `www.mathclass.org` by clicking homework scores on the main page.
We recommend to approach the web-based homework assignments via the following rules.

a) Start to work on an assignment as soon as the corresponding material is discussed in class.

b) Print out copies of your personal and of the common version (it is free in the Mathskeller, the student staff will show you how to do so) and put them in a notebook.

c) Get together with classmates to work on the problems via the printouts. The best thing is to work together on the common version. Write down the solutions in your notebook and only thereafter enter your solutions on the webpage. Check your answers by entering them into the system, and, if necessary, rework the problem and try to understand the correct answer provided to you by the system.

d) Thereafter work on the problems of your personal version and remember: only correct solutions to your personal version will earn you credit.

e) Bring the notebook with you when you go to office hours.

f) You are encouraged to discuss homework problems and the course material with each other. However, when it comes time for you to write up or enter the solutions, we expect you to do this completely on your own. It would be the best for your understanding if you put aside your notes from the discussions with your classmates and wrote up the solutions entirely from scratch.

g) If necessary, you may take the common version of the homework set with you to recitation and seek help.

h) If you feel you have worked a problem correctly and WHS marks it incorrect, please contact your teaching assistant or lecturer, for example, by e-mail.

2. Written Assignments:
These assignments are intended to help you learn to communicate mathematics and to present clear, well-written solutions to problems. Your solutions will be graded by humans for mathematical correctness and for clarity of exposition. Students who wish to receive full credit should write in complete, grammatically correct sentences. You should give clear reasoning and present the steps of your solution in logical order.

3. Optional homework:
There are various optional homework problems that do not count towards your grade: the

Goals:

In Calculus I, we will learn about derivatives, integrals and the fundamental theorems of calculus. We begin by introducing the notion of a limit. Limits are essential to defining derivatives and integrals. By the end of the semester students should know precise definitions of continuity, the derivative, and the integral and understand the fundamental theorem of calculus which relates the latter two. Students will apply the methods and ideas of calculus to solve physical and geometric problems.

We will cover most of Chapters 1 to 5 of Stewart's book. Please see the course calendar for a detailed listing of sections.

Exposure to the precision needed in Calculus will foster critical thinking and rational reasoning. In order to help you learn to formulate and communicate mathematical ideas, there will be six written assignments. Your solutions to these assignments are expected to be carefully drafted documents that are written up in complete sentences. You should lay out and explain all the arguments you used to arrive at your solution.

Grading

You can earn up to 500 points in the course based on the following activities:

3 exams	300 (100 points each)
Final exam	100
Homework and attendance	100
Total	**500**

The 100 points for homework and attendance are computed based on the following components:

Web homework	95 points
Written assignments	60 points (10 points each)
Attendance of the lectures	45 points
Total divided by 2	**100 points**

Your course grade will be based on the number of points you earn according to the following scheme:

Total earned course points (out of 500)	450-500	400-449	350-399	300-349	0-299
Final **course grade**	A	B	C	D	E

Students will be assigned a mid-term grade to allow them to gauge their performance at mid-term and, if necessary, make changes in their approach to learning calculus.

Homework and Quizzes:

There are three types of homework, details are described below; only the first two count towards the grade:

1. web-based homework,
2. 6 written assignments,
3. optional homework.

web-based assignments A0, AR, BR, CR, DR as well as optional homework assignments from the textbook, listed in the course calendar.
The optional assignment A0 is intended to introduce you to the syntax to enter mathematical expressions in the web homework system. The review assignments AR, BR, CR, and DR are study guides for each exam. All students are strongly advised to complete these review assignments and do optional homework from the textbook.

Quizzes will be given regularly during recitations (see the course calendar). The quizzes will not be collected and graded. They should help you to cope with a test situation where you have to work the given problems with closed books and a limited amount of time.

Late Homework:

No late submissions of web homework will be accepted. If an emergency or illness takes you away from school, please discuss your situation with your lecturer and ask to be excused from an assignment, if appropriate. If you have a scheduled absence (travel or authorized university absence) you must still submit the web homework by the deadline.

Written assignments are due at the beginning of the lecture. If an emergency or unexpected absence prevents you from turning in the assignment, please see your lecturer to request permission to turn in the assignment late. If you have a scheduled absence (travel or authorized university absence) you should arrange to turn in the paper before leaving school. Unexcused and late submissions will be penalized 10% if the paper is turned in late on the due date and an additional 20% for each day that it is late.

Attendance:

You are expected and strongly advised to attend all lectures and recitations.

Lecturers will take attendance beginning January 21. Your attendance score is based on the percentage of lectures you attend. You will receive full credit (45 points, see above) if you have at most 2 unexcused absences. Attendance in recitation is required for a passing grade for MA 193 (see above), and is strongly recommended for everybody. Recitations are the place where you have a chance to actively engage, work problems under guidance, seek assistance, and communicate with your peers and the instructor.

Calculators and Laptop Computers:

Students may use a graphing calculator on exams and homework. The use of machines with symbolic manipulation capabilities is not allowed during examinations. Thus, no TI-89's, TI-92's, no HP-48's or higher versions or laptop computers may be used on exams. Please talk to your lecturer if you have any questions as to whether a particular machine may be used on a test. We may clear the memory of calculators before or during an examination.
The use of laptop computers is not allowed during lectures.

Using the web homework system on mathclass.org:

In order to access `www.mathclass.org` do the following steps (Students who registered near the beginning of the semester should wait 24 hours after they registered for MA 113):

- Use a web browser Internet Explorer 8.0 (or later version) or Firefox 3.1 (or later version). If you use Internet Explorer you will need to have additional plug-ins installed for correct display of the mathematical formulas. On Mac use Firefox 3.1 (or later version), Safari will not work.
- Go to `http://www.mathclass.org` and click on **Login to WHS**. (Do not(!!!) follow the "Register in WHS" link!)
- Log in using your campus active directory account login and password. This is the account you use to access your myUK. Enter your login name as `ad\UserName` where `UserName` is your active directory login name and use your current active directory account password. The password is case-sensitive!
- Follow all the instructions until you see the class MA 113 showing up. Print out the homework set A0 as well as the common and your personal version of A1.
- After the first time, you will be able to simply login with your `ad\UserName` and your active directory account password and you will be connected to your account.

If you have difficulty logging in, you may find further instructions on `https://www.mathclass.org/mc/Postings.aspx?poId=658`. You may also visit the Mathskeller (CB 063) M-F from 9 am to 5 pm.

Students who choose to drop MA 113 must drop through the registrar's office. Dropping your registration at `www.mathclass.org` will have no effect on your official registration.

Students who switch sections of MA 113 during the add-drop period will have their registration at `www.mathclass.org` updated automatically. When a student changes sections of MA 113 with the registrar's office, the account and record of homework will be automatically transferred to the new section at `www.mathclass.org`.

Study Advice and Getting Help:

It is essentially impossible to passively teach mathematics; it must be actively learned. To understand what this means, consider the impossibility of learning to play tennis by listening to someone describe how to play tennis or by watching some world-class player. You will not learn the material in this course by just listening to the lectures, and thinking to yourself – "Yes, I understand that". You must work the problems and go through the difficulties yourself before you will begin to learn. The instructor's task is that of an assistant to help you learn as much of the material as you desire.

This being said, form good study skills from the start! Come to class. Read the text prior to the lecture where it will be covered. Take notes and **do the homework**. Find classmates to study with. Do not fall behind. It is very difficult to catch up in a math class after falling behind. **Use old exams of MA 113 to take a practice test by yourself in an exam-like situation. Compare your solutions with those provided by the answer key.** If you are having trouble, then seek help without delay.

If you are having trouble with a homework problem, you can send an e-mail through the online homework system to your teaching assistant. Try to provide as much information as possible in your help request.

If you need more help than what can be provided by the online help, you should take one or more of the following steps.

- Talk to your instructors before or after class or send them an email, if necessary. Let them know what problems you are having, if any. They will be happy to help!
- Go to the office hours of your instructors.
- You can also seek help in the **Mathskeller** that is located in room CB 063 in the basement of the classroom building. Many instructors and teaching assistants from the Department of Mathematics will hold office hours in the Mathskeller. In addition, limited drop-in tutoring is available. You can seek help from any of the instructors or teaching assistants — not just your own. The Mathskeller is open from 9 am to 5 pm Monday through Friday (except academic holidays) during the semester. Additional information is available at `www.mathskeller.org`.
- Furthermore, you can seek help in **The Study** located on the 3rd floor of the Commons, South Campus. Academic Enhancement provides drop-in peer tutoring by experienced undergraduate students who have successfully navigated the courses for which they tutor. A regular schedule of all tutoring is available on The Study's website `www.uky.edu/ugs/study`. You can also call 257-1356.

You can find more detailed suggestions of how to study for the course on the handout "Some Suggestions on How to Study Mathematics", see also `www.math.uky.edu/~heidegl/Ma113S10/Handouts/HowToSucceed.pdf`.

Policies:

1. Attend lectures and recitations regularly. Be on time and remain until dismissed. Do not leave in the middle of class. Instructors have the right to take off attendance points for coming late or leaving early.
   If you cannot come to lecture or recitation and would like to request an excused absence let the instructor know about it next time in class (see also the section on attendance).
2. Classes are cell phone-free and laptop-free zones! Cell phones and laptops must be off and out of sight for the entire class period (see also the section on calculators and laptop computers). Instructors have the right to take off attendance points for using cell phones or laptops during class. The same applies to reading newspapers or other activities unrelated to the course.
3. In order to be fair to all students, dates for exams and homework assignments are firm. It is very important to take each exam on schedule. Missed work may be made up only due to illness with medical documentation or for other unusual (documented) circumstances (see also the section on late homework). If you have a university excused absence or a university-scheduled class conflict with uniform examinations please contact your lecturer as soon as possible, **at least 10 days before the exam**, so that an alternate exam can be arranged for you. No alternate exam will be given more than 8 days after the common hour exam.
4. *Academic Honesty:* Students are encouraged to work together to understand a problem and to develop a solution. However, the solution you submit for credit must be your own work. In particular, you should write your solutions to the written assignments independently. Copying on exams and usage of books, notes, or communication devices during examinations is not allowed. Cheating or plagiarism is a serious offense, and it will not be tolerated. Students are responsible for knowing the University policy on cheating.

5. To earn top grade on exam problems and written assignments it is not enough to have the correct answer, but you must also show the correct reasoning.
6. Classes do meet as usual on the days after an exam and on Friday before Spring break. Attendance rules apply as usual.
7. *Accommodations due to disability:* If you have a documented disability that requires academic accomodations, please see your instructor as soon as possible during scheduled office hours. In order to receive accommodations in this course, you must provide your instructor with a letter of accommodation form the Disability Resource Center (Room 2, Alumni Gym, 257 2754, jkarnes@email.uky.edu) for coordination of campus disability services available to students with disabilities.

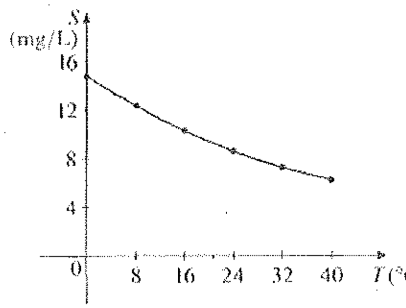
Calendar for Ma 113: Calculus - Spring 2010

Lecture *Recitation*	**In-Class Activities**	**Due Dates**	**Optional Textbook Problems**
13-Jan	1.1 - 1.4: Domain, Range; Linear and Quadratic Functions		p.74: 1,3,5,6,10,11,19; p. A15: 1,7,17,18,21,37; p. A23: 11,14,29,33
14-Jan	*Pretest, Worksheet 1*		
15-Jan	1.6: Inverse Functions (w/o Log and Inverse Trig)		1.6: 1-13 odd,21,27,29,33,35
18-Jan	Martin-Luther-King Day: Academic Holiday		
19-Jan	*Worksheet 2 (#1-6), Assignment 1 handed out*		
20-Jan	1.5, 1.6: Exp. and Log. Functions (w/o e and ln)	A1, **Last day to add a class**	1.5: 1-11 odd; 1.6: 33,35,37
21-Jan	*Worksheet 2 (#7-10), Quiz 1*		
22-Jan	2.1 The Tangent and Velocity Problem		2.1: 1,3,5,7
25-Jan	2.2 The Limit of a Function		2.2: 1,5,7,9,13,15,25,27,33
26-Jan	*Worksheets 3, 4*	A2	
27-Jan	2.3 Limit Laws	A3, Assgn1 due in class	2.3: 1-15 odd,21,25,29
28-Jan	*Worksheet 5, Quiz 2, Assignment 2 handed out*		
29-Jan	2.5 Continuity	A4	2.5: 3-13 odd,16,19,21,23,35,37,41,47
1-Feb	2.7 Derivatives and Rates of Change		2.7: 1-9 odd, 13,17,19,25,27,31
2-Feb	*Worksheet 6*	A5	
3-Feb	2.8 The Derivative as a Function	A6, **Last day to drop**	2.8: 1,3,5,9,19,23,25,35
4-Feb	*Worksheet 7, Quiz 3*		
5-Feb	Review	A7, Assgn2 due in class	
8-Feb	Review		
9-Feb	*Worksheet 8*		
9-Feb	**Exam 1, 7:30-9:30 PM, room TBA**		
10-Feb	3.1 Derivatives of Poly. and Exp. Fct's (introduce e and ln)	**Last day to withdraw for 50% refund**	3.1: 1,3,5,7,15,17,21,23,31,33,39,47
11-Feb	*Worksheet 9, Assignment 3 handed out*		
12-Feb	3.2 The Product and Quotient Rules	B1	3.2: 1,3,7,11,15,23,27
15-Feb	Appendix D and 1.6: Trig and Inverse Trig Functions		App D: 1,7,13,19,20,29,31,33,35,43,51,59,65; 1.6: 59,61,63,65
16-Feb	*Worksheet 10*		
17-Feb	3.3 Derivatives of Trig Functions	B2	3.3: 1,5,9,15,17,21,33,39
18-Feb	*Worksheet 11, Quiz 4*		
19-Feb	3.4 Chain Rule	B3, Assgn3 due in class	3.4: 1,5,9,19,23,35,47
22-Feb	3.5 Implicit Differentiation and Derivatives of Inverse Trig	B4	3.5: 1,5,11,19,21,27,33
23-Feb	*Worksheet 12, Assignment 4 handed out*		
24-Feb	3.6 Derivatives of Logarithms (w/o Logarithmic Diff'n)	B5	3.6: 3,7,13,19,33,37,43
25-Feb	*Worksheet 13, Quiz 5*		
26-Feb	3.7 Rates of Change	B6	3.7: 1,5,9,15,21,23
1-Mar	3.8 Exponential Growth and Decay	B7	3.8: 3,5,7,11,13
2-Mar	*Worksheet 14*		
3-Mar	3.9 Related Rates	B8	3.9: 3,7,13,15,25,31,37,43
4-Mar	*Worksheet 15, Quiz 6*		
5-Mar	Review	B9, Assgn4 due in class	

8-Mar	Review		
9-Mar	*Worksheet 16*		
9-Mar	**Exam 2, 7:30-9:30 PM, room TBA**		
10-Mar	4.1 Maximum and Minimum Values		4.1: 5,9,11,13,17,21,25,29,33,34,41,49,51,57,61
11-Mar	*Worksheet 17, Assignment 5 handed out*		
12-Mar	4.2 The Mean Value Theorem	C1	4.2: 3,5,7,11,15,19,23,25
15-Mar	Spring Break		
16-Mar	Spring Break		
17-Mar	Spring Break		
18-Mar	Spring Break		
19-Mar	Spring Break		
22-Mar	4.3 How Derivatives Affect the Shape of a Graph		4.3: 3,5,7,11,17,19,25,31
23-Mar	*Worksheet 18*		
24-Mar	2.6 Limits at Infinity, Horizontal Asymptotes	C2	2.6: 3,5,7,13,19,25,33,41,49,53(a)
25-Mar	*Worksheet 19, Quiz 7*		
26-Mar	4.4 L'Hopital's Rule (w/o Differences and Powers)	C3, Assgn5 due in class	4.4: 1,3,5,9,17,21,29,43,55
29-Mar	4.5 Summary of Curve Sketching (w/o Slant Asymptotes)		4.5: 5,9,17,19,33,41
30-Mar	*Worksheet 20, Assignment 6 handed out*		
31-Mar	4.7 Optimization Problems	C4	4.7: 3,5,11,13,17
1-Apr	*Worksheet 21, Quiz 8*		
2-Apr	4.7 Optimization Problems	C5, **Last Day to withdraw**	4.7: 19,33,55
5-Apr	3.10 Linear Appr. (w/o Differentials)		3.10:1,3,9,23,29; 4.8: 3,5,11,17,21,31,33
6-Apr	*Worksheet 22*	C6	
7-Apr	4.9 Anti-Derivatives		4.9: 3,7,15,21,23,31,39
8-Apr	*Worksheet 23, Quiz 9*	C7	
9-Apr	Review	C8, Assgn6 due in class	
12-Apr	Review		
13-Apr	*Worksheet 24*		
13-Apr	**Exam 3, 7:30-9:30 pm, room TBA**		
14-Apr	5.1 Areas and Distances		5.1: 3,11,15,17,21
15-Apr	*Worksheet 25*		
16-Apr	5.2 The Definite Integral	D1	5.2: 1,5,9,19,21,23,33,37,49,53,55
19-Apr	5.3 The Fundamental Theorem of Calculus		5.3: 3,5,9,13,17,19,27,31,39,51,53
20-Apr	*Worksheet 26*		
21-Apr	5.4 Indefinite Integrals and Net Change	D2	5.4: 3,5,9,15,23,31,37,43
22-Apr	*Worksheets 27, 28 (#1-4), Quiz 10*		
23-Apr	5.5 Substitution Rule	D3	5.5: 3,7,13,19,21,25,33,43,59,67,75
26-Apr	Review		
27-Apr	*Worksheets 28 (#5-7), 29*		
28-Apr	Review	D4	
29-Apr	*Worksheet 30*		
30-Apr	Review		
5-May	**Final exam, 6:00-8:00 PM, room TBA**		

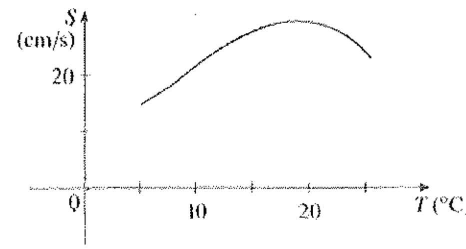
48. The quantity (in pounds) of a gourmet ground coffee that is sold by a coffee company at a price of p dollars per pound is $Q = f(p)$.
(a) What is the meaning of the derivative $f'(8)$? What are its units?
(b) Is $f'(8)$ positive or negative? Explain.

49. The quantity of oxygen that can dissolve in water depends on the temperature of the water. (So thermal pollution influences the oxygen content of water.) The graph shows how oxygen solubility S varies as a function of the water temperature T.
(a) What is the meaning of the derivative $S'(T)$? What are its units?
(b) Estimate the value of $S'(16)$ and interpret it.



Adapted from *Environmental Science: Science Lives Within the System of Nature*, 2d ed., by Charles E. Kupchella, © 1989. Reprinted by permission of Prentice-Hall, Inc., Upper Saddle River, NJ

50. The graph shows the influence of the temperature T on the maximum sustainable swimming speed S of Coho salmon.
(a) What is the meaning of the derivative $S'(T)$? What are its units?
(b) Estimate the values of $S'(15)$ and $S'(25)$ and interpret them.



51–52 Determine whether $f'(0)$ exists.

51. $f(x) = \begin{cases} x \sin \dfrac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

52. $f(x) = \begin{cases} x^2 \sin \dfrac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

WRITING PROJECT

EARLY METHODS FOR FINDING TANGENTS

The first person to formulate explicitly the ideas of limits and derivatives was Sir Isaac Newton in the 1660s. But Newton acknowledged that "If I have seen further than other men, it is because I have stood on the shoulders of giants." Two of those giants were Pierre Fermat (1601–1665) and Newton's teacher at Cambridge, Isaac Barrow (1630–1677). Newton was familiar with the methods that these men used to find tangent lines, and their methods played a role in Newton's eventual formulation of calculus.

The following references contain explanations of these methods. Read one or more of the references and write a report comparing the methods of either Fermat or Barrow to modern methods. In particular, use the method of Section 2.7 to find an equation of the tangent line to the curve $y = x^3 + 2x$ at the point $(1, 3)$ and show how either Fermat or Barrow would have solved the same problem. Although you used derivatives and they did not, point out similarities between the methods.

1. Carl Boyer and Uta Merzbach, *A History of Mathematics* (New York: Wiley, 1989), pp. 389, 432.
2. C. H. Edwards, *The Historical Development of the Calculus* (New York: Springer-Verlag, 1979), pp. 124, 132.
3. Howard Eves, *An Introduction to the History of Mathematics*, 6th ed. (New York: Saunders, 1990), pp. 391, 395.
4. Morris Kline, *Mathematical Thought from Ancient to Modern Times* (New York: Oxford University Press, 1972), pp. 344, 346.

Assignment 2 for MA 113 - Calculus I (Fall 2009)

September 10, 2009

Instructions: The purpose of this and subsequent assignments is to develop your ability to formulate and communicate a mathematical argument showing step-by-step reasoning.

Please give a complete, well-written solution to each of the following problems. Your work will be graded for accuracy, completeness, and grammatically correct English.

Your solutions should be neat and legible, stapled, and your name should appear on each sheet. Moreover, on page 1 of your solution, please also indicate your *section number* to insure that you will receive proper credit for the assignment.

Due Date: Your completed solutions are due on **Friday, September 18, 2009**, at the beginning of lecture.

(1) (3 Points) Let x be any real number except -4 and let $f(x) = \dfrac{3}{x+4}$. Find the limit

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

(the result will be a function of x).

For the following exercise, it will be useful to recall that we can rewrite the difference of radicals by multiplying and dividing by the conjugate:

$$\sqrt{a}-\sqrt{b} = \frac{(\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})}{\sqrt{a}+\sqrt{b}} = \frac{a-b}{\sqrt{a}+\sqrt{b}}.$$

(2) (6 Points) Consider the function $f(x) = \sqrt{25-x^2}$ and let a be a number in the open interval $(-5, 5)$.

(a) For any number x in $(-5, 5)$, determine the slope of the secant line through the points $(a, f(a))$ and $(x, f(x))$ and its limit L as x approaches a. Find the equation of the line through the point $(a, f(a))$ with slope L. This line is called the tangent line to the graph of f at the point $(a, f(a))$.

(b) Assume $a \neq 0$ and determine the slope of the line through the origin and the point $(a, f(a))$.

(c) Use a typical number a and plot the graph of f, the line through the origin and the point $(a, f(a))$, and the tangent line you computed in (a) on the same set of axes. Interpret your results from (a) and (b) geometrically.

(3) (1 Point) Write the definition of the derivative of a function f at a point a.

Bonus Problem: (2 Points)

A group of professors in the Math department go out for dinner. They have drinks and food and everything, and their agreement is to share the bill evenly. Just before the bill comes, two of the professors go to the bathroom, manage to climb out the window, and leave for good! The bill comes and it is a whopping \$180 (including tax and tip). Professor Lovely says: "Those guys walked out on us again! But look. If everyone, in addition to their original share of the bill, throws in an extra three bucks, we can exactly cover the bill." How many people were in the original group?

Before beginning, it might be helpful to recall the quadratic formula. The roots of the quadratic equation $ax^2 + bx + c = 0$ are

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

The quantity inside the radical, $b^2 - 4ac$, is called the *discriminant.* It is easy to see that we have two real roots if the discriminant is positive, one real root if the discriminant is 0 and no real roots if the discriminant is negative.

1. Find all tangent lines to the parabola $y = x^2$ that pass through the point $(0, -2)$.

2. Consider the parabola $y = x^2$, and a point (c, d) which may or may not lie on the parabola. We will determine how many tangent lines to the parabola pass through (c, d). The exercises below answer this question and allow you to relate the number of tangent lines to the location of the point.

   (a) For each of the points below, make a sketch which shows the parabola given by $y = x^2$ and all tangent line(s) to this parabola which pass through the specified point.

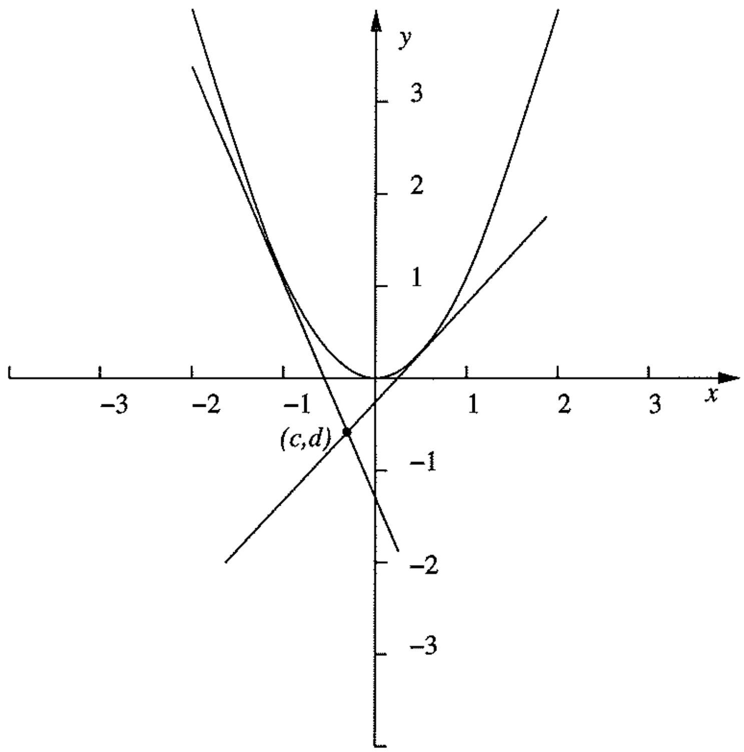
      i. $(1, -2)$
      ii. $(1, 1)$
      iii. $(0, 1)$.

   (b) Make a conjecture as to how many tangent lines of the parabola pass through a given point (c, d). How does the answer depend on the point (c, d)?

   (c) Write the equation of the tangent line to the parabola $y = x^2$ at (a, a^2).

   (d) If we require the tangent line in part c) to pass through point (c, d), we obtain an equation for a. Write out this equation. Solving this equation will give the x-coordinate of the point where the tangent line meets the parabola.

   (e) Give conditions on c and d which tell us that we have exactly 0, 1 or 2 tangent lines through (c, d). Interpret your answers geometrically. What do these conditions tell us about the location of the point (c, d)?



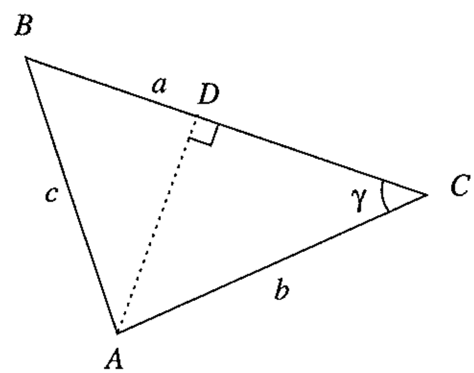
February 4, 2007

Worksheet 3. MA113
Calculus I Fall 2006

As always, your work should be written out neatly and carefully. Use complete sentences.

1. Consider a triangle with vertices A, B and C. The point D is on the side BC and the line segments AD and BC are perpendicular. Apply the Pythagorean theorem and the definition of the cosine function to show that if a, b and c are the lengths of the sides of the triangle and γ is the measure of the angle opposite the side of length c, then

$$c^2 = a^2 + b^2 - 2ab\cos\gamma.$$



2. Use the principle of mathematical induction to prove the differentiation rule for powers,

$$\frac{d}{dx}x^n = nx^{n-1}, \qquad n = 1, 2, 3, \ldots$$

Hint: The base case follows easily from the definition. For the induction step, write $x^{N+1} = x \cdot x^N$ and use the product rule.

3. Use the definition of the derivative to prove that if f is differentiable at a number a and $f(a) \neq 0$, then the reciprocal g defined by $g(x) = 1/f(x)$ is differentiable at a and

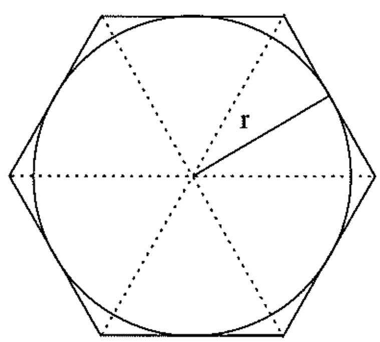
$$g'(a) = \frac{-f'(a)}{f(a)^2}.$$

In your paper, you should explain why f is continuous at a and why this is needed to find the derivative of g at a.

Written assignment 6. Optimization. MA113
Calculus I Spring 2007

Answer the following questions. Display your answers clearly and neatly. Explain your reasoning. Use complete sentences.

1. Let u and v be two numbers which are positive or zero, whose sum is 10 and so that u^2v is as large as possible.
   (a) Write down a function f and an interval $[a, b]$ so that the maximum value of f on the interval $[a, b]$ occurs at the number u described above.
   (b) Find u and v.
2. Suppose a circle of radius r is inscribed in a hexagon as pictured. Find the area of the hexagon. (This formula for the area of a hexagon will be needed in the next problem.)



3. Answer parts 1–2 of the project in Stewart, pages 288–89. It will be helpful to read the example 2 on page 279–280 before beginning the project.

March 8, 2007

(a) Use Poiseuille's Law to show that the total resistance of the blood along the path ABC is

$$R = C\left(\frac{a - b\cot\theta}{r_1^4} + \frac{b\csc\theta}{r_2^4}\right)$$

where a and b are the distances shown in the figure.

(b) Prove that this resistance is minimized when

$$\cos\theta = \frac{r_2^4}{r_1^4}$$

(c) Find the optimal branching angle (correct to the nearest degree) when the radius of the smaller blood vessel is two-thirds the radius of the larger vessel.

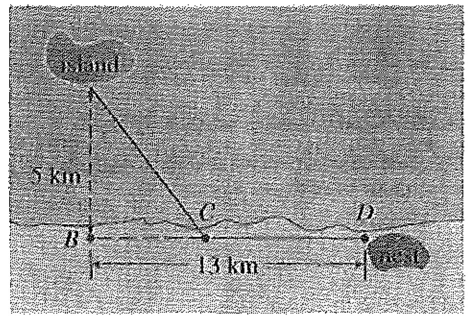
73. Ornithologists have determined that some species of birds tend to avoid flights over large bodies of water during daylight hours. It is believed that more energy is required to fly over water than land because air generally rises over land and falls over water during the day. A bird with these tendencies is released from an island that is 5 km from the nearest point B on a straight shoreline, flies to a point C on the shoreline, and then flies along the shoreline to its nesting area D. Assume that the bird instinctively chooses a path that will minimize its energy expenditure. Points B and D are 13 km apart.

(a) In general, if it takes 1.4 times as much energy to fly over water as land, to what point C should the bird fly in order to minimize the total energy expended in returning to its nesting area?

(b) Let W and L denote the energy (in joules) per kilometer flown over water and land, respectively. What would a large value of the ratio W/L mean in terms of the bird's flight? What would a small value mean? Determine the ratio W/L corresponding to the minimum expenditure of energy.

(c) What should the value of W/L be in order for the bird to fly directly to its nesting area D? What should the value of W/L be for the bird to fly to B and then along the shore to D?

(d) If the ornithologists observe that birds of a certain species reach the shore at a point 4 km from B, how many times more energy does it take a bird to fly over water than land?



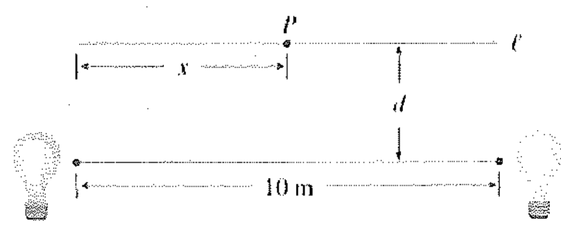
74. Two light sources of identical strength are placed 10 m apart. An object is to be placed at a point P on a line ℓ parallel to the line joining the light sources and at a distance d meters from it (see the figure). We want to locate P on ℓ so that the intensity of illumination is minimized. We need to use the fact that the intensity of illumination for a single source is directly proportional to the strength of the source and inversely proportional to the square of the distance from the source.

(a) Find an expression for the intensity $I(x)$ at the point P.

(b) If $d = 5$ m, use graphs of $I(x)$ and $I'(x)$ to show that the intensity is minimized when $x = 5$ m, that is, when P is at the midpoint of ℓ.

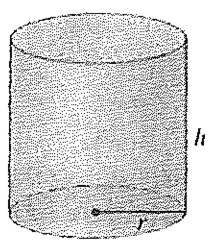
(c) If $d = 10$ m, show that the intensity (perhaps surprisingly) is *not* minimized at the midpoint.

(d) Somewhere between $d = 5$ m and $d = 10$ m there is a transitional value of d at which the point of minimal illumination abruptly changes. Estimate this value of d by graphical methods. Then find the exact value of d.



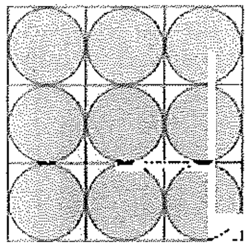
APPLIED PROJECT

THE SHAPE OF A CAN

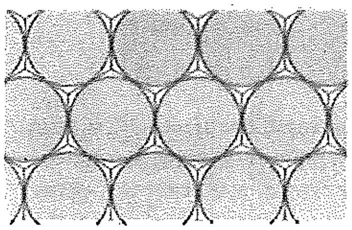


In this project we investigate the most economical shape for a can. We first interpret this to mean that the volume V of a cylindrical can is given and we need to find the height h and radius r that minimize the cost of the metal to make the can (see the figure). If we disregard any waste metal in the manufacturing process, then the problem is to minimize the surface area of the cylinder. We solved this problem in Example 2 in Section 4.7 and we found that $h = 2r$; that is, the height should be the same as the diameter. But if you go to your cupboard or your supermarket with a ruler, you will discover that the height is usually greater than the diameter and the ratio h/r varies from 2 up to about 3.8. Let's see if we can explain this phenomenon.

1. The material for the cans is cut from sheets of metal. The cylindrical sides are formed by bending rectangles; these rectangles are cut from the sheet with little or no waste. But if the



Discs cut from squares



Discs cut from hexagons

top and bottom discs are cut from squares of side $2r$ (as in the figure), this leaves considerable waste metal, which may be recycled but has little or no value to the can makers. If this is the case, show that the amount of metal used is minimized when

$$\frac{h}{r} = \frac{8}{\pi} \approx 2.55$$

2. A more efficient packing of the discs is obtained by dividing the metal sheet into hexagons and cutting the circular lids and bases from the hexagons (see the figure). Show that if this strategy is adopted, then

$$\frac{h}{r} = \frac{4\sqrt{3}}{\pi} \approx 2.21$$

3. The values of h/r that we found in Problems 1 and 2 are a little closer to the ones that actually occur on supermarket shelves, but they still don't account for everything. If we look more closely at some real cans, we see that the lid and the base are formed from discs with radius larger than r that are bent over the ends of the can. If we allow for this we would increase h/r. More significantly, in addition to the cost of the metal we need to incorporate the manufacturing of the can into the cost. Let's assume that most of the expense is incurred in joining the sides to the rims of the cans. If we cut the discs from hexagons as in Problem 2, then the total cost is proportional to

$$4\sqrt{3}\,r^2 + 2\pi rh + k(4\pi r + h)$$

where k is the reciprocal of the length that can be joined for the cost of one unit area of metal. Show that this expression is minimized when

$$\frac{\sqrt[3]{V}}{k} = \sqrt[3]{\frac{\pi h}{r}} \cdot \frac{2\pi - h/r}{\pi h/r - 4\sqrt{3}}$$

4. Plot $\sqrt[3]{V}/k$ as a function of $x = h/r$ and use your graph to argue that when a can is large or joining is cheap, we should make h/r approximately 2.21 (as in Problem 2). But when the can is small or joining is costly, h/r should be substantially larger.

5. Our analysis shows that large cans should be almost square but small cans should be tall and thin. Take a look at the relative shapes of the cans in a supermarket. Is our conclusion usually true in practice? Are there exceptions? Can you suggest reasons why small cans are not always tall and thin?

4.8 NEWTON'S METHOD

Suppose that a car dealer offers to sell you a car for \$18,000 or for payments of \$375 per month for five years. You would like to know what monthly interest rate the dealer is, in effect, charging you. To find the answer, you have to solve the equation

$$48x(1 + x)^{60} - (1 + x)^{60} + 1 = 0 \qquad \boxed{1}$$

(The details are explained in Exercise 41.) How would you solve such an equation?

For a quadratic equation $ax^2 + bx + c = 0$ there is a well-known formula for the roots. For third- and fourth-degree equations there are also formulas for the roots, but they are

Worksheet 7. MA113

Calculus I Fall 2006

As always, your work should be written out neatly and carefully. Use complete sentences.

1. Let $f(x) = \int_0^x \frac{1}{2+t^2}\,dt$. Find the interval(s) where f is increasing and the interval(s) where f is decreasing. Find the interval(s) where f is concave up and the interval(s) where f is concave down.

2. Suppose that f is a continuous function defined on the real line. Let $g(t) = \int_0^t f(s)\,ds$.

   (a) Compute $g'(t)$.

   (b) If $n > 0$, find an expression for $\frac{d}{dt}[(g(t))^{n+1}]$ and show that

   $$\int_0^x f(t)\left(\int_0^t f(s)\,ds\right)^n dt = \frac{1}{n+1}\left(\int_0^x f(s)\,ds\right)^{n+1}.$$

   Hint: Do not use the principle of mathematical induction. If F and G have the same derivative, what do we know about $F - G$?

3. The Great Pyramid at Giza is of height 135 meters and the base is a square with sides of length 230 meters. Follow the steps below to compute the volume in cubic meters of the Great Pyramid of Giza. You may check your answer using the formula $V = \frac{1}{3}Ah$ for the volume of a pyramid of height h and whose base has area A.

   (a) We view the pyramid as built up in layers starting with the base. Each layer is square slab with thickness Δy. Draw a picture of a pyramid and indicate a typical layer. Find an approximate value for the volume of a layer that is y meters above the ground and of thickness Δy. Give your answer in terms of y and Δy.

   (b) Pick a partition $0 = y_0 < y_1 < y_2 \ldots < y_n = 135$ which divides the pyramid into n slabs of equal thickness. Compute the volume of each slab and sum to obtain an approximation to the total volume of the pyramid.

   (c) As the thickness of each layer tends to zero, the sum approaches a definite integral. Find this integral and evaluate it to give the volume of the pyramid.

Worksheet 5. Rainbows. MA113
Calculus I Fall 2006

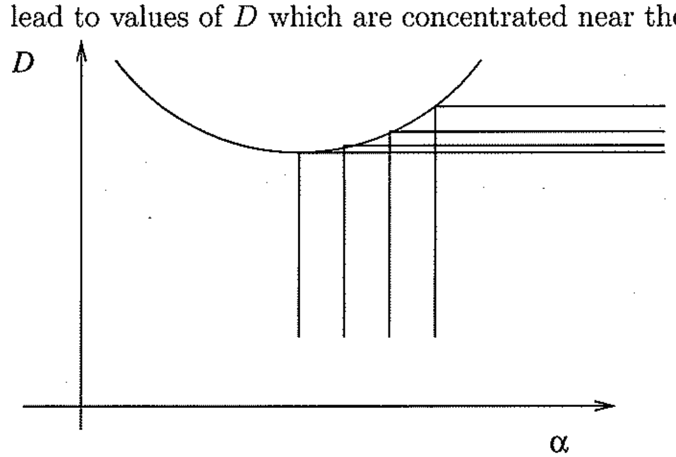
Write out your answers carefully and in complete sentences.

1. Let f be defined for all real numbers and suppose that $0 \leq f'(x) \leq 1$ for all x. Can we have $f(1) = 2$ and $f(4) = 6$? Find an example to show this is possible or give a careful explanation as to why it is impossible.

2. Carry out parts 1 and 2 of the project "The calculus of rainbows", page 232 of the fifth edition of Stewart. In part 1, you do not need to show that the critical number is a minimum. As we try to explain below, any critical number should lead to a concentration of light.

3. (1 point extra credit) The sky below a rainbow is often either brighter or darker than the sky above the rainbow. Is it brighter or darker? The critical number you found in part 1) is a minimum. Use that the critical number is a minimum and a sketch showing a few typical light rays passing through a raindrop to help explain whether it is brighter above or below the rainbow.

We have used critical numbers to help us find local extreme values for a function. This project shows another reason why the critical numbers of a function are important. If $f'(c) = 0$, then the linear approximation to f at c is a constant function (another name for a point where the derivative is zero is stationary point). When we see a rainbow in the sky, the rainbow is formed by light rays being concentrated near a critical point of the function $D(\alpha)$ discussed in this project. The drawing below helps to show why a rainbow corresponds to a critical number of $D(\alpha)$. The graph shows that near a critical number of D, equally spaced values of α, lead to values of D which are concentrated near the value of D at the critical point.



73. When a foreign object lodged in the trachea (windpipe) forces a person to cough, the diaphragm thrusts upward causing an increase in pressure in the lungs. This is accompanied by a contraction of the trachea, making a narrower channel for the expelled air to flow through. For a given amount of air to escape in a fixed time, it must move faster through the narrower channel than the wider one. The greater the velocity of the airstream, the greater the force on the foreign object. X rays show that the radius of the circular tracheal tube contracts to about two-thirds of its normal radius during a cough. According to a mathematical model of coughing, the velocity v of the airstream is related to the radius r of the trachea by the equation

$$v(r) = k(r_0 - r)r^2 \qquad \tfrac{1}{2}r_0 \leq r \leq r_0$$

where k is a constant and r_0 is the normal radius of the trachea. The restriction on r is due to the fact that the tracheal wall stiffens under pressure and a contraction greater than $\frac{1}{2}r_0$ is prevented (otherwise the person would suffocate).

(a) Determine the value of r in the interval $\left[\frac{1}{2}r_0, r_0\right]$ at which v has an absolute maximum. How does this compare with experimental evidence?

(b) What is the absolute maximum value of v on the interval?

(c) Sketch the graph of v on the interval $[0, r_0]$.

74. Show that 5 is a critical number of the function

$$g(x) = 2 + (x - 5)^3$$

but g does not have a local extreme value at 5.

75. Prove that the function

$$f(x) = x^{101} + x^{51} + x + 1$$

has neither a local maximum nor a local minimum.

76. If f has a minimum value at c, show that the function $g(x) = -f(x)$ has a maximum value at c.

77. Prove Fermat's Theorem for the case in which f has a local minimum at c.

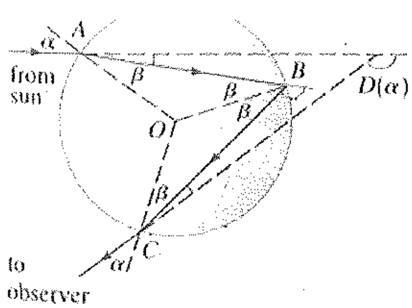
78. A cubic function is a polynomial of degree 3; that is, it has the form $f(x) = ax^3 + bx^2 + cx + d$, where $a \neq 0$.

(a) Show that a cubic function can have two, one, or no critical number(s). Give examples and sketches to illustrate the three possibilities.

(b) How many local extreme values can a cubic function have?

APPLIED PROJECT — THE CALCULUS OF RAINBOWS

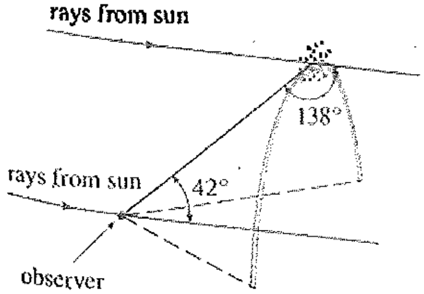
Rainbows are created when raindrops scatter sunlight. They have fascinated mankind since ancient times and have inspired attempts at scientific explanation since the time of Aristotle. In this project we use the ideas of Descartes and Newton to explain the shape, location, and colors of rainbows.



Formation of the primary rainbow

1. The figure shows a ray of sunlight entering a spherical raindrop at A. Some of the light is reflected, but the line AB shows the path of the part that enters the drop. Notice that the light is refracted toward the normal line AO and in fact Snell's Law says that $\sin\alpha = k\sin\beta$, where α is the angle of incidence, β is the angle of refraction, and $k \approx \frac{4}{3}$ is the index of refraction for water. At B some of the light passes through the drop and is refracted into the air, but the line BC shows the part that is reflected. (The angle of incidence equals the angle of reflection.) When the ray reaches C, part of it is reflected, but for the time being we are more interested in the part that leaves the raindrop at C. (Notice that it is refracted away from the normal line.) The *angle of deviation* $D(\alpha)$ is the amount of clockwise rotation that the ray has undergone during this three-stage process. Thus

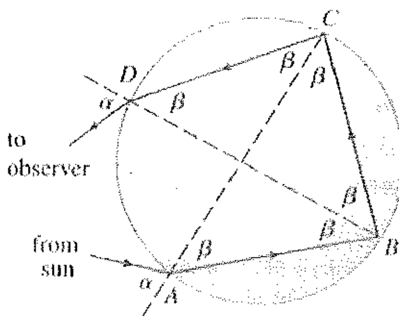
$$D(\alpha) = (\alpha - \beta) + (\pi - 2\beta) + (\alpha - \beta) = \pi + 2\alpha - 4\beta$$



Show that the minimum value of the deviation is $D(\alpha) \approx 138°$ and occurs when $\alpha \approx 59.4°$.

The significance of the minimum deviation is that when $\alpha \approx 59.4°$ we have $D'(\alpha) \approx 0$, so $\Delta D/\Delta\alpha \approx 0$. This means that many rays with $\alpha \approx 59.4°$ become deviated by approximately the same amount. It is the *concentration* of rays coming from near the direction of minimum deviation that creates the brightness of the primary rainbow. The figure at the left shows that the angle of elevation from the observer up to the highest point on the rainbow is $180° - 138° = 42°$. (This angle is called the *rainbow angle*.)

2. Problem 1 explains the location of the primary rainbow, but how do we explain the colors? Sunlight comprises a range of wavelengths, from the red range through orange, yellow, green, blue, indigo, and violet. As Newton discovered in his prism experiments of 1666, the index of refraction is different for each color. (The effect is called *dispersion.*) For red light the refractive index is $k \approx 1.3318$ whereas for violet light it is $k \approx 1.3435$. By repeating the calculation of Problem 1 for these values of k, show that the rainbow angle is about $42.3°$ for the red bow and $40.6°$ for the violet bow. So the rainbow really consists of seven individual bows corresponding to the seven colors.



Formation of the secondary rainbow

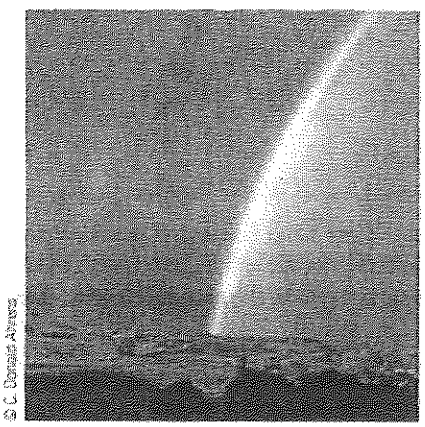
3. Perhaps you have seen a fainter secondary rainbow above the primary bow. That results from the part of a ray that enters a raindrop and is refracted at A, reflected twice (at B and C), and refracted as it leaves the drop at D (see the figure). This time the deviation angle $D(\alpha)$ is the total amount of counterclockwise rotation that the ray undergoes in this four-stage process. Show that

$$D(\alpha) = 2\alpha - 6\beta + 2\pi$$

and $D(\alpha)$ has a minimum value when

$$\cos\alpha = \sqrt{\frac{k^2 - 1}{8}}$$

Taking $k = \frac{4}{3}$, show that the minimum deviation is about $129°$ and so the rainbow angle for the secondary rainbow is about $51°$, as shown in the figure.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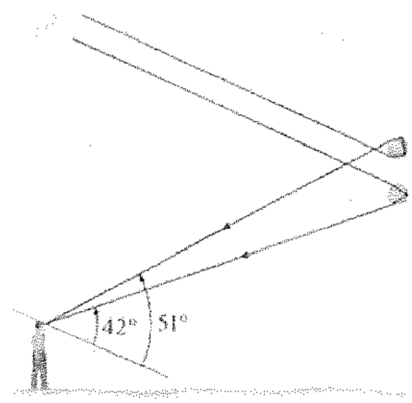


4. Show that the colors in the secondary rainbow appear in the opposite order from those in the primary rainbow.

4.2 THE MEAN VALUE THEOREM

We will see that many of the results of this chapter depend on one central fact, which is called the Mean Value Theorem. But to arrive at the Mean Value Theorem we first need the following result.

■ Rolle's Theorem was first published in 1691 by the French mathematician Michel Rolle (1652–1719) in a book entitled *Méthode pour résoudre les égalitéz*. He was a vocal critic of the methods of his day and attacked calculus as being a "collection of ingenious fallacies." Later, however, he became convinced of the essential correctness of the methods of calculus.

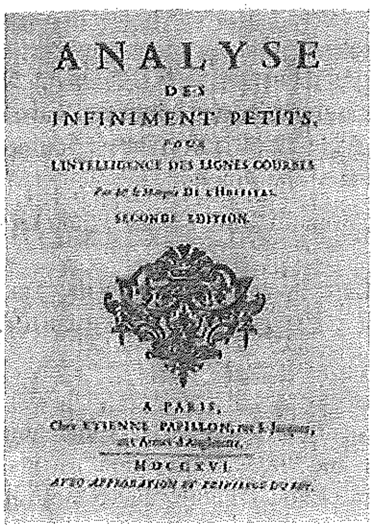
ROLLE'S THEOREM Let f be a function that satisfies the following three hypotheses:

1. f is continuous on the closed interval $[a, b]$.
2. f is differentiable on the open interval (a, b).
3. $f(a) = f(b)$

Then there is a number c in (a, b) such that $f'(c) = 0$.

WRITING PROJECT

THE ORIGINS OF L'HOSPITAL'S RULE



Thomas Fisher Rare Book Library

www.stewartcalculus.com
The Internet is another source of information for this project. Click on *History of Mathematics* for a list of reliable websites.

L'Hospital's Rule was first published in 1696 in the Marquis de l'Hospital's calculus textbook *Analyse des Infiniment Petits*, but the rule was discovered in 1694 by the Swiss mathematician John (Johann) Bernoulli. The explanation is that these two mathematicians had entered into a curious business arrangement whereby the Marquis de l'Hospital bought the rights to Bernoulli's mathematical discoveries. The details, including a translation of l'Hospital's letter to Bernoulli proposing the arrangement, can be found in the book by Eves [1].

Write a report on the historical and mathematical origins of l'Hospital's Rule. Start by providing brief biographical details of both men (the dictionary edited by Gillispie [2] is a good source) and outline the business deal between them. Then give l'Hospital's statement of his rule, which is found in Struik's sourcebook [4] and more briefly in the book of Katz [3]. Notice that l'Hospital and Bernoulli formulated the rule geometrically and gave the answer in terms of differentials. Compare their statement with the version of l'Hospital's Rule given in Section 4.4 and show that the two statements are essentially the same.

1. Howard Eves, *In Mathematical Circles (Volume 2: Quadrants III and IV)* (Boston: Prindle, Weber and Schmidt, 1969), pp. 20–22.
2. C. C. Gillispie, ed., *Dictionary of Scientific Biography* (New York: Scribner's, 1974). See the article on Johann Bernoulli by E. A. Fellmann and J. O. Fleckenstein in Volume II and the article on the Marquis de l'Hospital by Abraham Robinson in Volume VIII.
3. Victor Katz, *A History of Mathematics: An Introduction* (New York: HarperCollins, 1993), p. 484.
4. D. J. Struik, ed., *A Sourcebook in Mathematics, 1200–1800* (Princeton, NJ: Princeton University Press, 1969), pp. 315–316.

4.5 SUMMARY OF CURVE SKETCHING

So far we have been concerned with some particular aspects of curve sketching: domain, range, and symmetry in Chapter 1; limits, continuity, and asymptotes in Chapter 2; derivatives and tangents in Chapters 2 and 3; and extreme values, intervals of increase and decrease, concavity, points of inflection, and l'Hospital's Rule in this chapter. It is now time to put all of this information together to sketch graphs that reveal the important features of functions.

You might ask: Why don't we just use a graphing calculator or computer to graph a curve? Why do we need to use calculus?

It's true that modern technology is capable of producing very accurate graphs. But even the best graphing devices have to be used intelligently. We saw in Section 1.4 that it is extremely important to choose an appropriate viewing rectangle to avoid getting a misleading graph. (See especially Examples 1, 3, 4, and 5 in that section.) The use of calculus enables us to discover the most interesting aspects of graphs and in many cases to calculate maximum and minimum points and inflection points *exactly* instead of approximately.

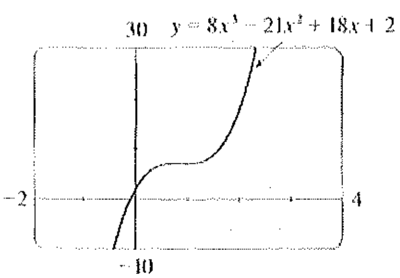


FIGURE 1

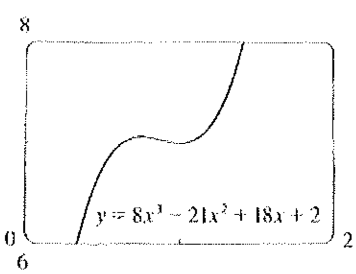
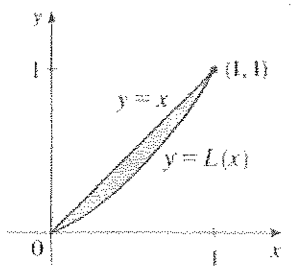


FIGURE 2

For instance, Figure 1 shows the graph of $f(x) = 8x^3 - 21x^2 + 18x + 2$. At first glance it seems reasonable: It has the same shape as cubic curves like $y = x^3$, and it appears to have no maximum or minimum point. But if you compute the derivative, you will see that there is a maximum when $x = 0.75$ and a minimum when $x = 1$. Indeed, if we zoom in to this portion of the graph, we see that behavior exhibited in Figure 2. Without calculus, we could easily have overlooked it.

In the next section we will graph functions by using the interaction between calculus and graphing devices. In this section we draw graphs by first considering the following households receive $a\%$ of the income, in which case the Lorenz curve would be the line $y = x$. The area between the Lorenz curve and the line $y = x$ measures how much the income distribution differs from absolute equality. The *coefficient of inequality* is the ratio of the area between the Lorenz curve and the line $y = x$ to the area under $y = x$.



(a) Show that the coefficient of inequality is twice the area between the Lorenz curve and the line $y = x$, that is, show that

$$\text{coefficient of inequality} = 2\int_0^1 [x - L(x)]\,dx$$

(b) The income distribution for a certain country is represented by the Lorenz curve defined by the equation

$$L(x) = \tfrac{5}{12}x^2 + \tfrac{7}{12}x$$

What is the percentage of total income received by the bottom 50% of the households? Find the coefficient of inequality.

68. On May 7, 1992, the space shuttle *Endeavour* was launched on mission STS-49, the purpose of which was to install a new perigee kick motor in an Intelsat communications satellite. The table gives the velocity data for the shuttle between liftoff and the jettisoning of the solid rocket boosters.

Event	Time (s)	Velocity (ft/s)
Launch	0	0
Begin roll maneuver	10	185
End roll maneuver	15	319
Throttle to 89%	20	447
Throttle to 67%	32	742
Throttle to 104%	59	1325
Maximum dynamic pressure	62	1445
Solid rocket booster separation	125	4151

(a) Use a graphing calculator or computer to model these data by a third-degree polynomial.

(b) Use the model in part (a) to estimate the height reached by the *Endeavour*, 125 seconds after liftoff.

WRITING PROJECT

NEWTON, LEIBNIZ, AND THE INVENTION OF CALCULUS

We sometimes read that the inventors of calculus were Sir Isaac Newton (1642–1727) and Gottfried Wilhelm Leibniz (1646–1716). But we know that the basic ideas behind integration were investigated 2500 years ago by ancient Greeks such as Eudoxus and Archimedes, and methods for finding tangents were pioneered by Pierre Fermat (1601–1665), Isaac Barrow (1630–1677), and others. Barrow—who taught at Cambridge and was a major influence on Newton—was the first to understand the inverse relationship between differentiation and integration. What Newton and Leibniz did was to use this relationship, in the form of the Fundamental Theorem of Calculus, in order to develop calculus into a systematic mathematical discipline. It is in this sense that Newton and Leibniz are credited with the invention of calculus.

Read about the contributions of these men in one or more of the given references and write a report on one of the following three topics. You can include biographical details, but the main thrust of your report should be a description, in some detail, of their methods and notations. In particular, you should consult one of the sourcebooks, which give excerpts from the original publications of Newton and Leibniz, translated from Latin to English.

The Role of Newton in the Development of Calculus

The Role of Leibniz in the Development of Calculus

- The Controversy between the Followers of Newton and Leibniz over Priority in the Invention of Calculus

References

1. Carl Boyer and Uta Merzbach, *A History of Mathematics* (New York: Wiley, 1987), Chapter 19.

2. Carl Boyer, *The History of the Calculus and Its Conceptual Development* (New York: Dover, 1959), Chapter V
3. C. H. Edwards, *The Historical Development of the Calculus* (New York: Springer-Verlag, 1979), Chapters 8 and 9.
4. Howard Eves, *An Introduction to the History of Mathematics*, 6th ed. (New York: Saunders, 1990), Chapter 11.
5. C. C. Gillispie, ed., *Dictionary of Scientific Biography* (New York: Scribner's, 1974). See the article on Leibniz by Joseph Hofmann in Volume VIII and the article on Newton by I. B. Cohen in Volume X.
6. Victor Katz, *A History of Mathematics: An Introduction* (New York: HarperCollins, 1993), Chapter 12.
7. Morris Kline, *Mathematical Thought from Ancient to Modern Times* (New York: Oxford University Press, 1972), Chapter 17

Sourcebooks

1. John Fauvel and Jeremy Gray, eds., *The History of Mathematics: A Reader* (London: MacMillan Press, 1987), Chapters 12 and 13.
2. D. E. Smith, ed., *A Sourcebook in Mathematics* (New York: Dover, 1959), Chapter V.
3. D. J. Struik, ed., *A Sourcebook in Mathematics, 1200–1800* (Princeton, N.J.: Princeton University Press, 1969), Chapter V.

5.5 THE SUBSTITUTION RULE

Because of the Fundamental Theorem, it's important to be able to find antiderivatives. But our antidifferentiation formulas don't tell us how to evaluate integrals such as

$$\boxed{1} \qquad \int 2x\sqrt{1+x^2}\,dx$$

To find this integral we use the problem-solving strategy of *introducing something extra*. Here the "something extra" is a new variable; we change from the variable x to a new variable u. Suppose that we let u be the quantity under the root sign in (1), $u = 1 + x^2$. Then the differential of u is $du = 2x\,dx$. Notice that if the dx in the notation for an integral were to be interpreted as a differential, then the differential $2x\,dx$ would occur in (1) and so, formally, without justifying our calculation, we could write

■ Differentials were defined in Section 3.10. If $u = f(x)$, then

$$du = f'(x)\,dx$$

$$\boxed{2} \qquad \int 2x\sqrt{1+x^2}\,dx = \int \sqrt{1+x^2}\,2x\,dx = \int \sqrt{u}\,du$$

$$= \tfrac{2}{3}u^{3/2} + C = \tfrac{2}{3}(x^2+1)^{3/2} + C$$

But now we can check that we have the correct answer by using the Chain Rule to differentiate the final function of Equation 2:

$$\frac{d}{dx}\left[\tfrac{2}{3}(x^2+1)^{3/2} + C\right] = \tfrac{2}{3}\cdot\tfrac{3}{2}(x^2+1)^{1/2}\cdot 2x = 2x\sqrt{x^2+1}$$

In general, this method works whenever we have an integral that we can write in the form $\int f(g(x))g'(x)\,dx$. Observe that if $F' = f$, then

$$\boxed{3} \qquad \int F'(g(x))g'(x)\,dx = F(g(x)) + C$$